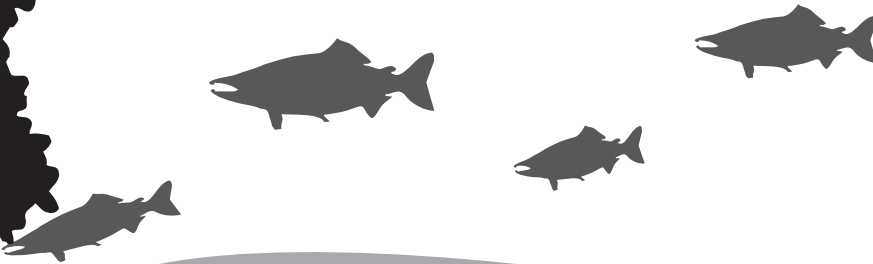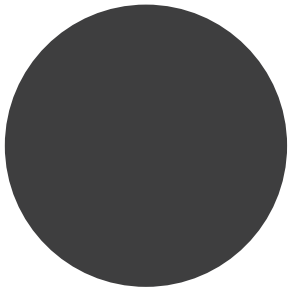# ADULT LITERACY FUNDAMENTAL MATHEMATICS

Book Six

ABE
ADULT BASIC EDUCTATION

Adult Literacy Fundamental Mathematics

Book 6

Prepared by Liz Girard

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To the Learner:

Welcome to Fundamental Mathematics Book Six.

Adult Math Learners

You have the skills you need to be a strong student in this class. Your instructor knows this because you have passed the Fundamental Math Level Five class, or you have been assessed into this level.

Adult math learners have many skills. They have a lot of life experience. They also use math in their everyday lives. This means that adult math learners may already know some of what is being taught in this book. Use what you already know with confidence!

Grades Record

You have also been given a sheet to write down your grades. After each test, you can write in the mark. This way you can keep track of your grades as you go through the course. This is a good idea to use in all your courses. You can find this grade sheet at the end of the book.

How to Use this Book

This textbook has:

- ✓ A **Table of Contents** listing the units, the major topics and subtopics.
- ✓ A **Glossary** giving definitions for mathematical vocabulary used in the course.
- ✓ A **grades record** to keep track of your marks.

The textbook has many exercises; some are quite short, but others have a great number of questions. **You do not have to do every single question!**

- Do as many questions as **you** feel are necessary for you to be confident in your skill.
- It is best to do **all** the word problems.
- If you leave out some questions, try doing every second or every third question. Always do some questions from the end of each exercise because the questions usually get harder at the end. You might use the skipped questions for review before a test.
- If you are working on a difficult skill or concept, do half the exercise one day and finish the exercise the next day. That is a much better way to learn.

Self-tests at the end of most topics have an **Aim** at the top. If you do not meet the aim, talk to your instructor, find what is causing the trouble, and do some more review before you go on.

A **Review and Extra Practice** section is at the end of each unit. If there is an area of the unit that you need extra practice in, you can use this. Or, if you want, you can use the section for more review.

A **Practice Test** is available for each unit. You may:

- Write the practice test after you have studied the unit as a practice for the end-of-chapter test,

  OR

- You might want to write it before you start the unit to find what you already know and which areas you need to work on.

Unit tests are written after each unit. Again, you must reach the **Aim** before you begin the next unit. If you do not reach the aim, the instructor will assist you in finding and practising the difficult areas. When you are ready, you can write a B test to show that you have mastered the skills.

A Final Test is to be written when you have finished the book. This final test will assess your skills from the whole book. You have mastered the skills in each unit and then kept using many of them throughout the course. The test reviews all those skills.

Math Anxiety

Math anxiety, or the fear of math, is something many people experience. It is a learned habit and can be unlearned. Math anxiety can happen for a few different reasons:

- Feeling anxious when writing tests
- Negative experiences in a past math class
- Embarrassment in a past math class
- Social pressures and expectations to not like math or not do well in math
- The want to get everything right
- Negative self-message ("I don't know how to do it", or "I hate math")

Everyone can learn math. There is no special talent people are born with that make them better at math. There are some people who are better at math than others, but even those people had to learn math to be good at it.

Do you suffer from math anxiety?

Read the list below and put a check mark beside the ones you feel when thinking about or doing math.

	Are your palms moist?
	Is your stomach fluttering?
	Do you feel like you can't think clearly?
	Do you feel like you would rather do anything else than learn math?
	Are you breathing faster than normal?
	Is your heart pounding?
	Do you feel cold?
	Do you feel sweaty?

If you answered yes to two or more of these items, you may have math anxiety.

How to Deal with Math Anxiety

Anyone can feel anxiety that will slow down learning. The key to learning is to be the "boss" of your anxiety.

One way to take control of your anxiety is to understand **test taking anxiety**.

There are four reasons people are anxious when writing tests. Any of the four reasons listed below might be the reason a person might feel anxious in a test-taking situation.

1) Not feeling prepared for the test
2) Not sure how to write the test in the best way
3) Feeling too much mental pressure
4) Poor health habits before writing a test

Here is an explanation of each reason and how to work your way out of the anxiety you may feel during tests.

1) Not feeling prepared for the test

Many students feel anxiety about taking math tests because they **do not feel prepared for the test.** To feel prepared, a student needs to have studied the work and know that they can do the problems they will be given. Get help from your classmates, friends or your instructor to find out how you can improve your study habits.

When you know a test is coming up, check to find out what is on the test and how long it will be. In this course, you have practice tests that look the same as the real test. This makes it easy to know what will be on the test. In future math courses, you may not get a practice test. You can ask your instructor for a test outline, which will give you a pretty good sense of what is on the test. Once you know all about what will be on the test, you can prepare for it better.

2) Not sure how to write the test in the best way

Here are some tricks students should know about how to write a test to do the best as possible on it:

Before the test:

a) **Arrive early**
   Get out all the supplies you need to do the test. (pencils, ruler, calculator, watch, etc)

b) **Be comfortable, but alert**
   Choose a good spot in the room and make sure you have enough space to work. Maintain a comfortable posture in your seat, but don't "slouch".

c) **Stay relaxed and confident**
   Keep a good attitude.
   If you find yourself anxious, take several slow, deep breaths to relax.
   Don't talk about the test to other students just before entering the room: their anxiety can be contagious.

Test Taking

a) **Read the directions carefully**
   This may be obvious, but it will help you avoid careless errors.

b) **Look through the test for an overview**
   Note key terms, jot down brief notes.
   Mark the test with comments that come to mind.

c) **Answer questions in a strategic order:**

   1. **Answer easy questions first**
      This will help to build confidence, and score points. It may also help you make connections with more difficult questions.

   2. **Then answer the difficult questions**
      Work on these harder questions with all the energy of the easier ones.

d) **Review**
   Resist the urge to leave as soon as you are done writing.
   Spend as much time as you can going over your test to see if you:
   o Answered all the questions
   o Wrote the answers in right
   o Did not make simple mistakes

3) Feeling too much mental pressure

There are many reasons why a student may feel mental pressure when writing a test. Listed below are a few main reasons:

- Negative beliefs about one's math abilities

- Low self-esteem when it comes to math

- Too high expectations of success

- Fear that failure or low grades will affect the future

- Feelings of pressure of not wanting to let down family members

When students feel this kind of pressure, it is very hard to feel calm and relaxed about a test. The key to success in a math test is to keep the anxiety at a manageable level. You can do this in two ways:

1. **Change negative self-talk.**

   Any time a negative thought creeps into your head, it will make it harder to stay positive and relaxed about your test.

   If you have a negative thought like "I can't do it", try to replace it with a positive thought like "I can do this".

2. **Use relaxing and calming techniques.**

   Use the calming breathing taught in earlier books of fundamental math. This will help you keep calm. Also, do not study in the last half hour before the test. You will be calmer by spending time relaxing and breathing deeply in that last half hour.

4) Poor health habits before writing a test

When your body and mind are healthy, you will have a better chance of doing well on a test. Eat well, drink plenty of water and get daily exercise. The better you feel, the better you can perform (and a test *is* a performance!)

Grades Record

Book 6

Unit	Practice Test	Date of Test A	Test A	Date of Test B	Test B
Example	√	*Sept. 4, 2011*	$\frac{25}{33}$	*Sept. 7, 2011*	$\frac{28}{33}$
1					
2					
3					
4					
Final Test					

Unit 1
Ratio, Rate
& Proportion

Introduction

Unit 1 will introduce ratios, rates and proportions.

- Using **ratios** we can compare quantities and see the relationship between the numbers in a simple way.

- **Rates** are used when a ratio is comparing two *different* kinds of measure.

- A **Proportion** is a statement with two equal ratios. They are be very useful in solving per cent problems

Unit 2 prepares you to work with per cents. You will learn to write **equivalent percents, decimals, and common fractions**.

Topic A: Writing Ratios

Ratio is a **comparison** of one number or quantity with another number or quantity. Ratio shows the **relationship** between the quantities.

> *Ratio* is pronounced "rā ' shō " or it can be pronounced "rā ' shē o ".
> You often use ratios, look at these examples:

- Scores in games are ratios. For example,
  - The Penguins won 4 to 3
  - The Canucks lost 1 to 5

- Directions for mixing can be ratios. For example,
  - Use 1 egg to each cup of milk
  - Mix 25 parts gas to 1 part oil for the motorcycle

- Betting odds are given as ratios. For example,
  - Black Jade is a 3 to 1 favourite
  - The heavyweight contender is only given a 2 to 5 chance to win

- The *scale* at the bottom of maps is a ratio. For example,
  - 1 centimetre represents 10 kilometres

- Prices are often given as ratios. For example,
  - 100 grams for $0.79
  - 2 cans for $1.85

For ratios to have meaning you must know **what** is being compared, and the units that are being used. Read these examples of ratios and the units that are used. A general ratio may say "parts" for the units.

A. It rained four days and was sunny for three days last week. The **ratio of rainy days to sunny days** was 4:3.

  4:3 is properly read "**4 is compared to 3**" but is often read "**4 to 3**".

B. The class has 12 men and 15 women registered. The ratio of men to women in the class is 12:15.

C. At the barbeque, 36 hot dogs and 18 hamburgers were eaten. The ratio of hot dogs eaten to hamburgers eaten is 36:18.

D. The class spends 3 hours on English and 2 hours on math each day. The ratio of time spent on English compared to math is 3:2.

Exercise One

Write the ratios asked for in these questions, using the : symbol (eg., 4:1). Write the units and what is being compared beside the ratio.

a) Powdered milk is mixed using 1 part of milk powder to 3 parts of water. Write a ratio to compare the milk powder to the water.

1:3   1 part of milk powder to 3 parts of water

b) One kilogram of ground beef will make enough hamburger for 5 people. Write a ratio to express the amount of ground beef for hamburgers to the number of people.

_____

c) Seventy-five vehicles were checked by the police. 15 vehicles did not meet the safety standards, but 60 of them did. Write a ratio comparing the unsafe vehicles to the safe vehicles.

_____

d) The liquid fertilizer we use when watering our house plants uses 7 drops of fertilizer to 1 litre of water. Write the ratio of fertilizer to water.

_____

e) The recipe says to roast a turkey according to its weight. For every kilogram, allow 40 minutes of cooking. Write a ratio comparing time to weight.

_____

f) The 4 litre pail of semi-transparent oil stain should cover 24 square metres of the house siding if the wood is smooth. Write the ratio comparing quantity of stain to the smooth wood surface area.

_____

g) The same 4 L of stain will only cover 16 square metres of the house siding if the wood is rough. Write that ratio.

_____

Answers to Exercise One

a) 1:3   1 part of milk powder to 3 parts water
b) 1:5   1 kg of beef to 5 people
c) 15:60   15 unsafe vehicles to 60 safe vehicles
d) 7:1   7 drops fertilizer to 1 litre of water
e) 40:1   40 minutes to 1 kg of turkey
f) 4:24   4 L of stain to 24 m^2 of smooth wood
g) 4:16   4 L of stain to 16 m^2 of rough wood

The numbers that you have been using to write the ratios are called the **terms of the ratio**.

The order that you use to write the terms is very important. You read a ratio from left to right and the order must match what the numbers mean. For example, 3 scoops of coffee to 12 cups of water must be written 3:12 as a ratio because you are comparing the quantity of coffee to the amount of water.

If you wish to talk about the amount of water compared to the coffee you have, you would say "use 12 cups of water for every 3 scoops of coffee" and the ratio would be written 12:3.

Ratios can be written 3 different ways:

- using the : symbol   2:5

- as a common fraction $\frac{2}{5}$
    - the first number in the ratio is the numerator; the second number is the denominator.
    - ratios written as a common fraction are read as a ratio, not as a fraction say "2 to 5", not two-fifths.
- using the word "to"   2 to 5

Exercise Two   Use the ratios you wrote in Exercise One to complete the chart.

	:	Common fraction	to
a)	1:3	$\frac{1}{3}$	1 to 3
b)			
c)			
d)			
e)			
f)			
g)			

Answers to Exercise Two

a) 1: 3 **or** $\frac{1}{3}$ **or** 1 to 3   b) 1: 5 **or** $\frac{1}{5}$ **or** 1 to 5   c) 15: 60 **or** $\frac{15}{60}$ **or** 15 to 60

d) 7: 1 **or** $\frac{7}{1}$ **or** 7 to 1   e) 40:1 **or** $\frac{40}{1}$ **or** 40 to 1   f) 4:24 **or** $\frac{4}{24}$ **or** 4 to 24

g) 4:16 **or** $\frac{4}{16}$ **or** 4 to 16

Exercise Three

Use the diagrams to write a ratio comparing the quantity of each shape, as asked.

a)

○	■	■	□	○
○	□	○	■	○
□	□	○	○	□
○	○	■	○	□
□	○	■	□	
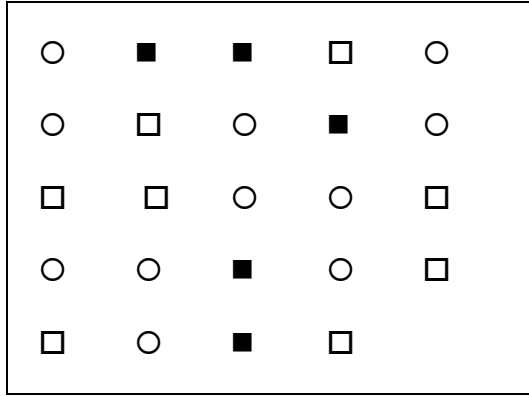
i) □ to ○   _____

ii) ■ to □   _____

iii) ○ to □   _____

iv) ■ to ○   _____

b)

✿	♥	☺	☺	☺
✿	♥	♥	✿	☺
♥	☺	✿	✿	☺
☺	♥	☺	✿	♥
☺	✿			
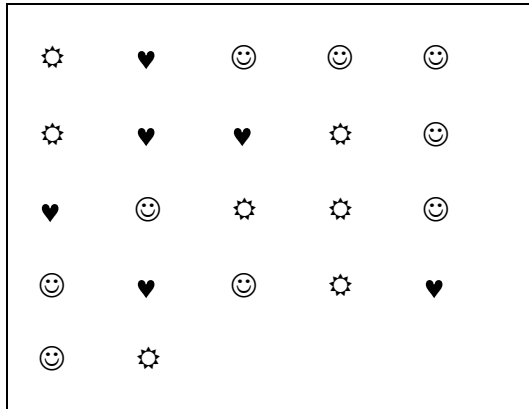
i) ☺ to ♥   _____

ii) ✿ to ☺   _____

iii) ♥ to ☺   _____

iv) ✿ to ♥ to ☺ _____

Answers to Exercise Three

a) i) 8:11 **or** $\frac{8}{11}$ **or** 8 to 11   ii) 5:8 **or** $\frac{5}{8}$ **or** 5 to 8   iii) 11:8 **or** $\frac{11}{8}$ **or** 11 to 8   iv) 5:11 **or** $\frac{5}{11}$ **or** 5 to 11

b) i) 9:6 **or** $\frac{9}{6}$ **or** 9 to 6   ii) 7:9 **or** $\frac{7}{9}$ **or** 7 to 9   iii) 6:9 **or** $\frac{6}{9}$ **or** 6 to 9   iv) 7:6:9 **or** 7 to 6 to 9

Equivalent Ratios

Like equivalent fractions, equivalent ratios are equal in value to each other.

$$10:100 = 1:10$$

Ratios can be written as common fractions. It is convenient to work with ratios in the common fraction form.

You can then easily:

- Find equivalent ratios in higher terms

- Find equivalent ratios in lower terms

- Find a missing term

Example A: Express 4:5 in higher terms

$$4:5 = \frac{4}{5} \longrightarrow \frac{4}{5}\left(\frac{\times 2}{\times 2}\right) \longrightarrow \frac{8}{10}$$

4:5 is equivalent to 8:10

Example B: Express 3:6 in lower terms

$$3:6 = \frac{3}{6} \longrightarrow \frac{3}{6}\left(\frac{\div 3}{\div 3}\right) \longrightarrow \frac{1}{2}$$

3:6 is equivalent to 1:2

To find equivalent ratios in higher terms, multiply each term of the ratio by the same number.

To find equivalent ratios in lower terms, divide each term of the ratio by the same number.

Exercise Four

Write equivalent ratios in any higher term. You may want to write the ratio as a common fraction first. Ask your instructor to mark this exercise.

a) $5:6 = \dfrac{5}{6}\left(\dfrac{\times 3}{\times 3}\right) = \dfrac{15}{18} = 15:18$

b) $4:3$ _____

c) $10:2$ _____

d) $50:1$ _____

e) $9:4$ _____

f) $3:5$ _____

g) $6:2$ _____

h) $7:8$ _____

i) $3:1$ _____

j) $1:4$ _____

Answers to Exercise Four: See your instructor.

Exercise Five

Write these ratios in lowest terms—that is, simplify the ratios.

a) $4:12 = \dfrac{4}{12}\left(\dfrac{\div 4}{\div 4}\right) = \dfrac{1}{3} = 1:3$

b) $10:5$ _____

c) $7:21$ _____

d) $20:5$ _____

e) 6:14 _____   f) 2:4 _____

g) 6:3 _____   h) 16:8 _____

i) 100:50 _____   j) 4:8 _____

Exercise Six

Using a colon : write a ratio in lowest terms for the information given.

a) In the class of 25 students, only 5 are smokers. Write the ratio of smokers to non-smokers in the class. (Note—you must first calculate the number of non-smokers.)

_____

b) The police issued 12 roadside suspensions to drivers out of the 144 who were checked in the road block last Friday. Write the ratio of suspended drivers to the number checked.

_____

c) Twenty-seven students registered for the course and 24 completed it. Write a ratio showing number of completions compared to number enrolled.

_____

d) During an hour (60 minutes) of television viewing last night there were 14 minutes of commercials, so there were only 46 minutes of the actual program! Write the ratio of commercial time to program time.

_____

e)   For each pair of coins, write the ratio comparing the value. (Use cents.)

   i)   a nickel to a dime   *5:10 = 1:2*

   ii)   a nickel to a quarter   _____

   iii)   a nickel to a dollar   _____

   iv)   a dime to a nickel   _____

   v)   a dime to a quarter   _____

   vi)   a dime to a dollar   _____

   vii)   a dollar to a dime   _____

Answers to Exercise Six

a) 1:4   b) 1:12   c) 8:9   d) 7:23
e) i) 1:2   ii) 1:5   iii) 1:20   iv) 2:1   v) 2:5   vi) 1:10   vii) 10:1

Topic A: Self-Test

A. Write the definitions. 3 marks

a) ratio _____

b) terms of the ratio _____

c) equivalent ratios _____

B. Write the ratios asked for in lowest terms. Use the : style. 4 marks
   Then *read* the ratio.

   a) The campground had three vacant campsites and 47 occupied sites. Write the ratio
       of occupied sites to vacant sites.

   ratio: _____   read: "_____."

   b) For every ten dogs in the city, only 2 have current dog licences. Write the ratio of
       licensed dogs to unlicensed dogs. (Find the number of unlicensed dogs first.)

   ratio: _____   read: "_____."

C. Simplify these ratios. 5 marks

a) 9:12 _____   b) 6:4 _____

c) 500:1 000 _____   d) 2:9 _____

e) 35:15 _____

Answers to Topic A Self-Test

Part A
a) A **ratio** is a comparison of one number or quantity with another number or quantity. Ratios show the relationship between the quantities or amounts.
b) **Terms of a ratio** are the numbers used in the ratio, the **parts** of the ratio.
c) **Equivalent ratios** – Ratios of equal value to each other.

Part B
a) 47:3   read "47 occupied sites to 3 vacant sites"
b) 1:4   read "1 licensed dog to 4 unlicensed dogs"

Part C
a) 3:4   b) 3:2   c) 1:2   d) 2:9   e) 7:3

Topic B: Rates

When a ratio is used to compare two different kinds of measure (e.g. apples and oranges, or meters and hours), it is called a rate. The denominator must be 1.

Example A:

A car can drive 725 km on 55 L of gas. What is the rate in km per L?

The ratio of this is $\dfrac{725km}{55L}$, find the rate by making the denominator 1.

Divide $\dfrac{725}{55}\left(\dfrac{\div 55}{\div 55}\right) = \dfrac{13.18}{1} = 13.18$

The rate is 13.18 km/L

Example B:

Sue bought 10 lb of oranges for $4.99. What is the rate in cents per pound?

The ratio is $\dfrac{\$4.99}{10lb} = \dfrac{499cents}{10lb}$, find the rate by making the denominator 1.

Divide $\dfrac{499}{10}\left(\dfrac{\div 10}{\div 10}\right) = \dfrac{49.9}{1} = 49.9$

The rate is 49.9 ¢/lb

When talking about rate, use the word 'per'.

In example A, say: "The fuel economy of the car is 13.18 kilometres per litre".

In example B, say: "The oranges cost 49.9 cents per pound".

Example C:

It takes 60 ounces of grass seed to plant 30 m^2 of lawn. What is the rate in ounces per metre squared (m^2)?

The ratio is $\dfrac{60oz}{30m^2}$, find the rate by making the denominator 1.

$$\frac{60}{30}\left(\frac{\div 30}{\div 30}\right) = \frac{2}{1} = 2$$

The rate is 2 oz/m^2, or

2 ounces per metre squared.

Exercise One

Write the following ratios as rates, comparing distance to time.

1) 120 km, 3 hours

2) 27 km, 9 hours

3) 203 km, 29 seconds

4) 444 km, 48 sec

Answers to Exercise One

1) 40 km/hour   2) 3 km/hour   3) 7 km/second   4) 9.25 m/second

Exercise Two

Write the following ratios as rates.

1) A leaky faucet can lose 52 litres of water in a week. What is the rate of litres lost per <u>day</u>? (round to two decimal places)

2) A ratio of distance travelled to time is called speed. What is the rate (speed) in kilometres per hour (km/h)?

a) 45 km, 3 hours   _____

b) 129 km, 1.5 hours   _____

c) 65 km, 13 hours   _____

3) Vancouver Island has a population of 734 860, and a land mass of 32 134 square kilometres. What is the rate of number of people per square kilometre? (This is called population density) Round your answer to the nearest whole number.

4) The population of PEI is 135 851, and its land mass is 5 660 square kilometres. What is the population density? Round your answer to the nearest whole number.

5) At rest, the human heart will beat an average of 4200 beats in 60 minutes. What is the rate of beats per minute?

6) At rest, the heart beat of an elephant is 1680 beats per 60 minutes. What is the rate of beats per minute?

7) At rest, the heart beat of a mouse is 30 000 beats per 60 minutes. What is the rate of beats per minute?

Answers to Exercise Two

1. 7.43 L/day   2. a. 15 km/hour   b. 86 km/hour   c. 5 km/hour
3. 23 people/km^2 4. 24 people/km^2 5. 70 beats/minute   6. 28 beats/minute   7. 500 beats/minute

Topic B: Self-Test

A. Write the definition.   1 mark

   a) Rate _____

               _____

               _____

B. Write the following ratios as rates.   6 marks

   a)   12 cups water, 3 cups sugar   _____

   b)   72 metres, 24 seconds   _____

   c)   1 365 000 people, 4000 km^2 _____

   d)   5000 cars on the road, 250 bikes on the road   _____

   e)   12 cups of flour, 12 tsp. of baking powder   _____

   f)   8 litres of gas, 2 litres of oil   _____

Answers to Self-Test

1. A rate is used when a ratio compares two different kinds of measure, and when the denominator is 1.

2. a) 4 water/sugar

   b) 3 m/second

   c) 341 people/km^2

   d) 20 cars/bike

   e) 1 cup flour/tsp baking powder

   f) 4 litres gas/litre oil

Topic C: Proportion

A proportion is a statement that two ratios are equal or equivalent. Here are some proportions:

Proportion	Fraction Form	Read like this...
1:2 = 2:4	$\frac{1}{2} = \frac{2}{4}$	1 is to 2 as 2 is to 4
1:4 = 25:100	$\frac{1}{4} = \frac{25}{100}$	1 is to 4 as 25 is to 100
18:9 = 10:5	$\frac{18}{9} = \frac{10}{5}$	18 is to 9 as 10 is to 5
15:20 = 3:4	$\frac{15}{20} = \frac{3}{4}$	15 is to 20 as 3 is to 4

Proportions can be used to solve many math problems. You will soon learn to use proportions to solve problems involving percent. The techniques you practice in the next few pages are important for that problem solving work.

Problems often give information that is an incomplete proportion; that is, one of the terms is missing. To solve such problems, you first find the comparison or ratio that is given. It may be:

- a quantity of one thing that is mixed with a larger quantity of something else.

- a scale of measurement given on a map such as 1 cm on the map represents 100 km distance on land.

- cost for a certain number of items.

- time to travel a certain distance.

The problem will then give one term of the second ratio in the proportion. For example, if you have been told that 3 heads of lettuce cost $1.49, you may be asked to find the cost of 7 heads of lettuce.

The missing term is the second cost. The proportion will be

$$\frac{\text{number of heads of lettuce}}{\text{cost}} = \frac{\text{number of heads of lettuce}}{\text{cost}}$$

$$\frac{3}{\$1.49} = \frac{7}{?}$$

$$3:\$1.49 = 7:?$$

The most important thing to remember is to keep the order of comparison the same in the first and second ratios in a proportion. If the first ratio compares time to distance then the second ratio in the proportion must compare time to distance.

$$\frac{\text{time}}{\text{distance}} = \frac{\text{time}}{\text{distance}}$$

or it could be

$$\frac{\text{distance}}{\text{time}} = \frac{\text{distance}}{\text{time}}$$

Once you have decided on the order of comparison it is a simple matter to write the proportion using the numbers given in the problem. Use a letter to stand for the missing term.

How would you find a missing term?

- You can use your skills with equivalent ratios (finding higher and lower terms).

- You can use your fraction skills of cross multiplying and then dividing to find the missing term.

Using Equivalent Ratios to Solve Proportions

Step 1 Decide on the order of comparison and write a ratio that describes the information given in the problem. Write a proportion using words of the items that are being compared in fraction form.

Step 2 Write two more ratios with the numbers matching the words in the first ratio. The missing term (number) can be given a letter (ex. N).

Step 3 Mentally set the ratio with words (the first ratio) aside.

Step 4 Multiply or divide the complete ratio to find the missing term.

Example A:

Use 1 teaspoon of baking powder for every 2 cups of flour. If a recipe uses 6 cups of flour, how much baking powder is needed?

The missing term is the teaspoons of baking powder for 6 cups of flour. Call this term N.

Step 1   Ratio is $\dfrac{\text{baking powder}}{\text{flour}}$

Step 2 $\dfrac{\text{baking powder}}{\text{flour}} = \dfrac{1}{2} = \dfrac{N}{6}$

Step 3 $\dfrac{1}{2} = \dfrac{N}{6}$

Step 4 $\dfrac{1}{2}\left(\dfrac{\times 3}{\times 3}\right) = \dfrac{3}{6}$ so $\dfrac{1}{2} = \dfrac{3}{6}$ so $N = 3$

Use 3 teaspoons of baking powder for 6 cups of flour.

Example B:

Reports suggest that 3 out of 10 people will at some time miss work due to back pain. If a company has 1000 employees, how many can be expected to miss work due to back pain.

The missing term is the number of people out of 1000 who will miss work due to back pain. Call this term P.

Step 1 $\dfrac{\text{people who miss work}}{\text{all the people at work}}$

Step 2 $\dfrac{\text{people who miss work}}{\text{all the people at work}} = \dfrac{3}{10} = \dfrac{P}{1000}$

Step 3 $\dfrac{3}{10} = \dfrac{P}{1000}$

Step 4 $\dfrac{3\,(\times 100)}{10\,(\times 100)} = \dfrac{300}{1000}$ so $\dfrac{3}{10} = \dfrac{300}{1000}$ so $P = 300$

300 people out of 1 000 people may miss work due to back pain.

Exercise One   Write the ratio of the words to describe the information given.

a) Three cup of flour to one teaspoon of yeast.

    a. Example: $\dfrac{\text{flour}}{\text{yeast}}$

b) Four parts oil, ten parts gasoline _____

c) One centimetre represents 100 kilometres _____

d) 100 grams for \$6.89 _____

e) 3 eggs for each cup of milk _____

f) 5 men and 7 women _____

g) Four hours spent on cleaning and two hours spent on making food. _____

Exercise Two   Use equivalent ratios to find the answers.

a) One cup of sugar and four cups of water will make great hummingbird food. How much sugar do you need for 8 cups of water?

$$\dfrac{sugar}{water} \longrightarrow \dfrac{1}{4} = \dfrac{N}{8} \longrightarrow \dfrac{1}{4}\left(\dfrac{\times 2}{\times 2}\right) = \dfrac{2}{8} \longrightarrow N = 2$$

b) Reports show that for every 100 vehicles checked by police, 20 vehicles do not meet the safety standard. If only 50 vehicles are checked, how many would not meet the safety standard?

c) Four litres of paint covers 24 square metres of wall. How much paint is needed to cover 72 square metres?

d) Powdered milk uses 1 part milk powder to 3 parts water. How much powder should be added to 9 parts water?

e) Five dollars will buy two litres of milk. How much milk can be bought with $7.50?

f) The Echinacea bottle directs to add 7 drops of Echinacea to 250 ml of water. How many drops go into 1 litre of water? (note: 1L = 1000 ml)

Answers to Exercise Two

a) 2 cups of sugar   b) 10 cars would not meet the safety standards   c) 12 litres of paint

d) 3 parts milk powder   e) 3 litres of milk   f) 28 drops of Echinacea

Exercise Three

Use equivalent ratios to find the missing term in these proportions.

a) $3:5 = Y:15$ _____

b) $1:2 = P:8$ _____

c) $5:7 = 10:N$ _____

d) $2:3 = 8:W$ _____

e) $4:7 = 16:A$ _____

f) $1:3 = 2:N$ _____

g) $9:10 = C:50$ _____

h) $500:1000 = N:2$ _____

i) $15:20 = 3:A$ _____

j) $9:11 = 18:N$ _____

k) The KX 250 motorcycle uses a mixture of one part oil to 30 parts of gasoline. How much oil must be added to 3000 mL (3 litres) of gasoline?

Answers to Exercise Three

a) Y = 9	b) P = 4	c) N = 14	d) W = 12
e) A = 28	f) N = 6	g) C = 45	h) N = 1
i) A = 4	j) N = 22	k) 1:30 = N:3000 N = 100 mL	

Using Cross-Multiplication to Solve a Proportion

Review Cross Products:

$$\frac{2}{5} \times \frac{4}{10} \qquad 2 \times 10 = 5 \times 4$$

$$20 \quad = \quad 20 \qquad \text{so, } \frac{2}{5} = \frac{4}{10}$$

Remember that when the cross products are the same, the fractions are equivalent.

When finding the missing terms in a proportion, cross-multiplication can be used. Follow the examples carefully.

Example A: $\qquad \frac{2}{3} = \frac{N}{45}$

Cross-multiply $\qquad 2 \times 45 = 3 \times N$

$$90 = 3N$$

The idea is to have the unknown term N by itself on one side of the equal sign. To do that, remember these things that you already know:

- division and multiplication are opposite operations

- whatever is done to one side of an equation or proportion must be done to the other side to keep the equation equal

3*N* means *N* is multiplied by 3. **To get rid of the 3, divide by 3.**

<u>You must also divide the other side of the equation by 3.</u>

$$\frac{90}{3} = \frac{3N}{3}$$

Solve by reducing the $\frac{3}{3}$ and by dividing 90 by 3.

$$\frac{90}{3} = \frac{3N}{3}$$

$$\frac{90}{3} = \frac{1N}{1}$$

$$\frac{90}{3} = N$$

$$90 \div 3 = N$$

$$30 = N$$

Reducing the fraction $\frac{3N}{3}$ to $\frac{1N}{1}$ to *N* is also called *cancelling*. In math, a fraction can be cancelled when the numerator and denominator are the same number.

$$\text{e.g.: } \frac{6P}{6} = \frac{1P}{1} = P$$

Example B: $6:7 = 24:N$

$$\frac{6}{7} = \frac{24}{N}$$

Cross-multiply $\quad 6 \times N = 7 \times 24$
$$6N = 168$$

Divide both sides by 6. The 6's with the N will cancel (reduce), and the N will be alone.

$$\frac{\cancel{6}N}{\cancel{6}} = \frac{168}{6}$$

$N = 168 \div 6$
$N = 28$

$$\frac{6}{7} = \frac{24}{28}$$

Check by cross-multiplying

Is $6 \times 28 = 7 \times 24$?

$6 \times 28 = 168$

$7 \times 24 = 168$

the cross-product $168 =$ the cross product 168

Yes $6:7 = 24:28$

Example C: $8:10 = N:80$

$$\frac{8}{10} = \frac{N}{80}$$

Cross-multiply $\quad 8 \times 80 = 10 \times N$
$$640 = 10N$$

Divide both sides by 10 so N will be alone.

$$\frac{640}{10} = \frac{10N}{10}$$

$$64 = N$$

To Solve a Proportion Problem Using Cross-Multiplication

Step 1 Write the first ratio using the information given.

Step 2 Write the proportion, using a letter in place of the missing term. Be sure the **order of comparison is the same** in both the first and second ratios in your proportion.

Step 3 Write the proportion in the fraction form. (Try to **simplify** the ratio **before** you do all the calculations.)

Step 4 Cross-multiply and set the cross-products equal to each other.

Step 5 Divide both sides of the equation by the number with the unknown term.

Step 6 Check by putting your answer back into the original proportion and cross-multiplying.

Exercise Four

Practise using cross-multiplying to find the missing term in these proportions.

a) $\dfrac{5}{8} = \dfrac{N}{32}$

$5 \times 32 = 8 \times N$

$160 = 8N$

$\dfrac{160}{8} = \dfrac{8N}{8}$

$160 \div 8 = N$

$20 = N$

b) $\dfrac{4}{N} = \dfrac{24}{30}$

c) $\dfrac{12}{4} = \dfrac{18}{x}$

d) $\dfrac{y}{6} = \dfrac{20}{12}$

e) $4{:}15 = 8{:}N$ f) $W{:}100 = 6{:}50$

g) $20{:}N = 5{:}4$ h) $5{:}12 = 25{:}x$

i) $6{:}9 = y{:}3$ j) $16{:}B = 8{:}3$

Answers to Exercise Four

b) $N = 5$	c) $x = 6$	d) $y = 10$	e) $N = 30$
f) $W = 12$	g) $N = 16$	h) $x = 60$	i) $y = 2$
j) $B = 6$			

The numbers in a ratio often are common fractions, decimals or mixed numbers. Follow exactly the same steps that you have been using to solve whole number proportions. The calculations will use your skills with fractions.

Example A: $2\frac{1}{4} : 3 = N:7$

Rewrite the proportion $\dfrac{2\frac{1}{4}}{3} = \dfrac{N}{7}$

Cross-multiply

$$2\frac{1}{4} \times 7 = 3 \times N$$

$$\frac{9}{4} \times \frac{7}{1} = 3 \times N$$

$$\frac{63}{4} = 3N$$

$$\frac{63}{4} \div \frac{3}{1} = \frac{3 \times N}{3} \qquad \longrightarrow \qquad \frac{63}{4} \times \frac{1}{3} = N$$

$$\frac{63}{12} = N \qquad \longrightarrow \qquad 5\frac{1}{4} = N$$

Exercise Five   Solve these proportions.

a) $6.5:5 = 13:A$

$\dfrac{6.5}{5} = \dfrac{13}{A}$

$6.5A = 65$

$A = 65 \div 6.5$

$A = 10$

b) $3\frac{1}{2} : 2 = N:8$

c) $9{:}6 = 4\frac{1}{2} : N$

d) $7.5{:}B = 10{:}20$

e) $3.75{:}5 = 9{:}x$

f) $4\frac{1}{8} : A = 3{:}6$

g) $Y : 1\frac{1}{2} = 12{:}4$

h) $6.125 : 24.5 = 4{:}N$

Answers to Exercise Five

b) N = 14   c) N = 3   d) B = 15   e) x = 12

f) A = $8\frac{1}{4}$ or 8.25   g) Y = $4\frac{1}{2}$ or 4.5   h) N = 16

Exercise Six

Solve these problems by proportion. Remember that the **order of comparison** must be the same in both ratios of a proportion.

a) Joanne can walk 18 km in 3 hours. How far can she walk, at the same rate in $5\frac{1}{2}$ hours?

b) The taxes on the property valued at $300 000 are valued at $5000. At the same rate of taxation, what would the taxes be on the smaller lot down the street which is valued at $240 000?

c) Liquid fish fertilizer smells awful but does a wonderful job! The concentrated fertilizer is mixed 15 millilitres to 4 litres of water. How much fertilizer would you mix with the 10 litre watering can of water?

d) One B.C. road map has a scale of 0.5 centimetres equal to 10 kilometres. Complete the chart by calculating actual driving distances in kilometres between some B.C. places.

The proportions will be $\dfrac{0.5}{10} = \dfrac{\text{cm given in chart}}{\text{actual distance in km}}$

Places in B.C.	Number of cm between places on the map	Actual distance in kilometres
Kelowna and Vernon	2.5 cm	
Burns Lake and Vanderhoof	5.5 cm	
TaTa Creek and Skookumchuk	0.75 cm	
Kitimat and Terrace	3.3 cm	

e) The directions on the lawn fertilizer say to spread 1.7 kg over 100 m^2 of lawn.

  i) How much fertilizer is needed for a 130 m^2 lawn?

  ii) How much fertilizer for a 75 m^2 lawn?

f) To mix concrete, combine 1 part Portland cement, 5 parts aggregate (a sand and gravel mixture) and $\frac{1}{2}$ part water. People often use a shovel for 1 part.

You want to use 3 shovels (parts) of cement.

    i) How much aggregate will you need?

    ii) How much water will you need?

Topic C: Self-Test

A. Solve these proportions. **6 marks**

a)   N:14 = 28:56

b)   3:11 = N:22

c)   50:45 = 10:N

d) $4\frac{1}{5}$:Y = 3:2

e)   12:4 = N:7

f)   7.75:N = 12:6

B. **14 marks**

a) Get a map of BC, a map of Canada and a map of your city or town.

b) Find the scale on each map (usually at the bottom) and write down the ratio of map distance to the actual distance.

c) With another student or an instructor, calculate actual distances between places by measuring the distance on the map and working out the proportion according to the scale given. Do at least three distance calculations on each map.

Ask your instructor to mark your work.

Answers to Topic C Self-Test

Part A

a) $N = 7$ b) $N = 6$ c) $N = 9$ d) $Y = 2\frac{4}{5}$ or 2.8

e) $N = 21$ f) $N = 3.875$

Part B – See your instructor.

Unit 1 Review

1. Write the ratios asked for in the questions. Reduce when needed.

    a) It snowed for two days and was sunny for five days last week. Write a ratio to compare snowy days to sunny days.

    b) The class spends 6 hours a week on math and 8 hours on English. Write the ratio of hours spent on math to English.

    c) The chickens at Glen's farm laid 3 eggs on Tuesday and 5 eggs on Friday. Write the ratio of numbers of eggs laid.

    d) The perimeter of a room is 14 metres, and the length is 3.5 metres. Write a ratio comparing the two numbers.

    e) One kilogram of pork will make enough sausages for 7 people. Write a ratio to express the amount of pork for sausages to the number of people.

    f) A jam recipe calls for 8 cups of blue berries to 3 cups of sugar. Write a ratio to express the amount of berries to sugar.

2. Write the ratios in lowest terms.

   a) 2:5 _____   b) 20:5 _____

   c) 14:21 _____   d) 2:4 _____

   e) 8:24 _____   f) 15:150 _____

   g) 130:26 _____   h) 9:54 _____

   i) 6:3 _____   j) 6:24 _____

3. Write the following ratios as rates.

   a) 16 cups of water to 4 cups of sugar   _____

   b) 150 kilometres to 1.25 hours   _____

   c) 12 cups of flour to 3 eggs   _____

   d) 465 000 people to 3000 square kilometres _____

   e) 35 kilometres to 7 hours   _____

   f) $3.99 to 10 pounds of apples   _____

4. Use cross multiplication to solve the proportions.

a) $\dfrac{5}{8} = \dfrac{P}{24}$

b) $\dfrac{S}{100} = \dfrac{8}{25}$

c) $\dfrac{4}{N} = \dfrac{8}{40}$

d) $\dfrac{4}{17} = \dfrac{F}{68}$

d) $\dfrac{8}{30} = \dfrac{12}{N}$

e) $\dfrac{6}{9} = \dfrac{A}{36}$

5. Use cross multiplication to solve the proportions.

a) $\dfrac{2\frac{1}{4}}{5} = \dfrac{A}{30}$

b) $\dfrac{L}{7\frac{1}{2}} = \dfrac{40}{30}$

c) $\dfrac{11}{8} = \dfrac{4\frac{1}{8}}{P}$

e) $\dfrac{5}{34} = \dfrac{Y}{1\frac{1}{2}}$

d) $\dfrac{3\frac{3}{4}}{5} = \dfrac{8}{X}$

f) $\dfrac{2\frac{2}{3}}{3} = \dfrac{A}{16}$

Answers to Unit 1 Review

1
a) 4:3   b) 3:4   c) 3:5   d) 14:3.5

e) 1 kg:7people   f) 8 cups berries:3 cups sugar

2
a) 1:3   b) 4:1   c) 2:3   d) 1:2

e) 1:3   f) 1:10   g) 5:1   h) 1:6

i) 2:1   j) 1:4

3
a) 4 cups water per cup of sugar   b) 120 km/hour

c) 4 cups flour per egg   d) 155 people per km^2

e) 5 km/hour   f) $0.399/pound or 39.9¢/pound

4
a) 15   b) 32   c) 20   d) 16
e) 45   f) 24

5
a) $13\frac{1}{2}$ b) 10   c) 3   d) $10\frac{2}{3}$
e) $10\frac{1}{5}$ f) $14\frac{2}{9}$

TEST TIME!

Ask your instructor for the Practice Test for this unit.

Once you've done the practice test,
you need to do the unit 1 test.

Again, ask your instructor for this.

Good luck!

Unit 2

Percent

Topic A: Introducing Percent

Percents are another form of fractions and are used in many everyday situations. Interest rates, credit card charges, taxes, pay deductions, increases and decreases are all calculated with percent. **Percents** are a **convenient** way to express part of the whole thing because the **unwritten denominator is always 100**.

- the denominator of 100 is not written or said.

- a percent sign **%** follows the number and is read as "percent".

- the whole thing is 100%

- **100% = 1**

> Remember that *cent* is the Latin word meaning a *hundred* **or** a *hundredth*. And *per* is a Latin word meaning *by* **or** *for each*. So, *percent* **means** *for each hundred*.

Reading and Writing Percents

To write a percent

- write the number in the usual way

- place the percent sign after the numerals

    50%

    $5\frac{1}{2}\%$ or 5.5%

    $\frac{3}{4}\%$ or 0.75%

To read a percent

- read the numbers in the usual way

- say "percent" after the number

    16% say "sixteen percent"

    $4\frac{1}{2}\%$ say "four and one-half percent"

    0.25% say "twenty-five hundredths percent" , "one-quarter percent"
    or "point two five percent".

Exercise One

Write these percents using numerals and a percent sign. Note that the mixed numbers may be expressed with common fractions or decimals.

a) thirty-four percent   _34%_

b) twelve percent   _____

c) four-fifths percent   _____

d) one hundred sixteen and three-tenths percent   _____

e) thirteen percent   _____

f) six and one-fifth percent   _____

g) ninety-four and one-half percent   _____

h) one-tenth percent   _____

i) one hundred percent   _____

j) one hundred fifty percent   _____

Answers to Exercise One

b) 12 %   c) 0.8 % or $\frac{4}{5}$ %   d) 116.3 % or $116\frac{3}{10}$ %

e) 13 %   f) 6.2 % or $6\frac{1}{5}$ %   g) 94.5 % or $94\frac{1}{2}$ %

h) 0.1 % or $\frac{1}{10}$ %   i) 100 %   j) 150 %

Exercise Two

Write the words to show how you say these percents.

a) 62%   _sixty-two percent_

b) $37\frac{1}{2}$ %   _____

c) 202%   _____

d) $\frac{3}{4}$ %   _____

e) $1\frac{1}{4}$ %   _____

f) 18.3%   _____

g) 14%   _____

h) 0.5%   _____

i) $\frac{1}{2}$ %   _____

j) 25%   _____

k) 100%   _____

Answers to Exercise Two

b) thirty-seven and one-half percent

c) two hundred two percent

d) three-quarters percent

e) one and one-quarter percent

f) eighteen and three-tenths percent

g) fourteen percent

h) five-tenths percent or one-half percent or zero point five percent

i) one-half percent

j) twenty-five percent

k) one hundred percent

Changing Decimals to Percents

Writing equivalent fractions is an important math skill.

Equivalent common fractions, decimals, and percents all represent the same amount.

$$\frac{1}{2} \ = \ 0.5 \ = \ 50\%$$

$$\frac{3}{10} \ = \ 0.3 \ = \ 30\%$$

You need the skill of writing equivalent fractions for working with percents.

> To change any number to a percent, multiply the number by 100% and place a percent sign **%** after the product.

Remember this shortcut for multiplying by 100?

$4.27 \times 100 = 427$

$0.287 \times 100 = 28.7$

$53 \times 100 = 5300$

The shortcut is: When multiplying by 100, **move the decimal point two places to the right**.

Example A: Change these numbers to a percent.

1	$1 \times 100\%$	$=$	100%
0.25	$0.25 \times 100\%$	$=$	25%
0.8	$0.8 \times 100\%$	$=$	80%
0.375	$0.375 \times 100\%$	$=$	37.5%

So...

<div style="border:1px solid black; padding:10px;">

To change a decimal to a percent, move the decimal point two places to the right and then write the percent sign after the number.

</div>

Examples: Change each decimal to a percent.

$$0.125 = 0.12.5 = 12.5\%$$

$$1.375 = 1.37.5 = 137.5\%$$

If the decimal point moves to the end of the number it is not necessary to write the decimal point. Remember that zeros at the beginning of a number are also not necessary.

$$0.24 = 0.24. = \cancel{0}24\% = 24\%$$

$$0.05 = 0.05. = \cancel{0}\cancel{0}5\% = 5\%$$

If the decimal is a tenth (one decimal place), it will be necessary to add a zero. If you are changing a whole number to a percent, add two zeros.

$$0.4 = 0.40. = 40\%$$

$$1.7 = 1.70. = 170\%$$

$$2 = 2.00. = 200\%$$

Exercise Three

Change these decimals to percents.

	Decimal	× 100% Move decimal 2 places to right	=	Percent
a)	0.75	0.75.	=	75%
b)	0.33			
c)	0.1			
d)	0.0025			
e)	0.07			
f)	0.9			
g)	0.166			
h)	0.43			
i)	0.325			
j)	0.088			
k)	0.0625			
l)	2.75			
m)	1.36			
n)	0.001			
o)	0.68			
p)	3			
q)	0.688			
r)	1.9			

Answers to Exercise Three

b) 33%	c) 10 %	d) 0.25 %	e) 7 %
f) 90 %	g) 16.6 %	h) 43 %	i) 32.5 %
j) 8.8 %	k) 6.25 %	l) 275 %	m) 136 %
n) 0.1 %	o) 68 %	p) 300 %	q) 68.8 %
r) 190 %			

Changing Percents to Decimals

Review **Dividing by 100** :

$$47.39 \div 100 = 0.4739$$

$$429 \div 100 = 4.29$$

$$3.824 \div 100 = 0.03824$$

To **divide by 100**, **move the decimal point two places to the left**.

Examples: Change each percent to a decimal or mixed number.

$$58\% \ = 58 \div 100 = \ \ .58 = \ \ \ \ 0.58$$
$$20\% \ = 20 \div 100 = \ \ \ .2 = \ \ \ \ 0.2$$
$$\ \ 6\% \ = \ \ 6 \div 100 = \ \ .06 = \ \ \ \ 0.06$$
$$110\% =110 \div 100 = \ 1.10$$

So...

> To change a percent to a decimal, divide by 100 (move the decimal point two places to the **left)** and remove the percent sign.

Examples: Change each percent to a decimal .

75%	=	75.0%	=	0.75	=	0.75
12%	=	12.0%	=	0.12	=	0.12
37.5%	=	37.5%	=	37.5%	=	0.375
125%	=	125.0%	=	1.25	=	1.25
5%	=	5.0%	=	0.05	=	0.05
4.6%	=	4.6%	=	0.046	=	0.046

Some notes to remember:

- If there is no decimal point in the percent, place the decimal point **after** the last numeral and **then** divide by 100.

    $24\% = 24.\% = 0.24$

- It may be necessary to **prefix** zeros. (This means adding zeros in front of the number, if needed)

    $6\% = 6.\% = 0.06$

- A zero at the right of a decimal is not needed and may be left off.

    $40\% = 40.\% = .40 = 0.4$

Exercise Four

Change each percent to its decimal equivalent.

	Percent	÷ 100 Move decimal 2 places to left	=	Decimal
a)	23%	.23.	=	0.23
b)	1%			
c)	9.2%			
d)	112%			
e)	10.3%			
f)	36%			
g)	75%			
h)	100%			
i)	8.25%			
j)	9%			
k)	14%			
l)	5%			
m)	4.4%			
n)	147%			
o)	11.5%			
p)	18%			
q)	4%			
r)	200%			

To change **a percent containing a common fraction** to a decimal, do this:

- Change the common fraction in the percent to a decimal in the percent.

- Divide by 100 (move the decimal 2 places to the **left**).

Examples:

$$3\frac{1}{2}\% \quad = 3.5\% \qquad\qquad 3.5\% \div 100 \ = \ .035 = 0.035$$

$$37\frac{1}{2}\% \ = 37.5\% \qquad\qquad 37.5\% \div 100 \ = .375 = 0.375$$

$$\frac{1}{4}\% \quad\quad = 0.25\% \qquad\qquad 0.25\% \div 100 \ = \ 0.0025$$

$$17\frac{1}{3}\% \quad = 17.\overline{3}\% \qquad\qquad 17.\overline{3}\% \ \div 100 = 0.17\overline{3}$$

Exercise Five

Change each percent to its decimal equivalent.

a) $8\frac{4}{5}\% =$ *8.8% =0.088*

b) $4\frac{1}{2}\% =$ _____

c) $56\frac{3}{4}\% =$ _____

d) $1\frac{3}{5}\% =$ _____

e) $112\frac{1}{2}\% =$ _____

f) $2\frac{3}{8}\% =$ _____

g) $5\frac{1}{4}\% =$ _____

h) $1\frac{1}{2}\% =$ _____

i) $2\frac{1}{8}\% =$ _____

j) $77\frac{1}{2}\% =$ _____

Answers to Exercise Five

b) 0.045   c) 0.5675   d) 0.016   e) 1.125
f) 0.02375   g) 0.0525   h) 0.015   i) 0.02125
j) 0.775

Changing Common Fractions to Percents

> To change any number to a percent, multiply the number by 100% and place the percent sign **%** after the product.

There are two methods you can use to change a common fraction to a percent.

Method One:

To change a common fraction to an equivalent percent, **multiply the common fraction by 100%**.

Examples:

$\dfrac{3}{4} = \underline{\quad}\%$
$\qquad \dfrac{3}{4} \times 100\% = \dfrac{3}{\cancel{4}_1} \times \dfrac{\cancel{100}^{25}}{1} = 75\%$

$\dfrac{5}{8} = \underline{\quad}\%$
$\qquad \dfrac{5}{8} \times 100\% = \dfrac{5}{\cancel{8}_2} \times \dfrac{\cancel{100}^{25}}{1} = \dfrac{125}{2}\% = 62.5\%$

$\dfrac{7}{10} = \underline{\quad}\%$
$\qquad \dfrac{7}{10} \times 100\% = \dfrac{7}{\cancel{10}_1} \times \dfrac{\cancel{100}^{10}}{1} = 70\%$

$1\dfrac{1}{5} = \underline{\quad}\%$
$\qquad 1\dfrac{1}{5} \times 100\% = \dfrac{6}{\cancel{5}_1} \times \dfrac{\cancel{100}^{20}}{1} = 120\%$

Exercise Six

Multiply by 100% to change each common fraction to an equivalent percent.

a) $\frac{4}{5}$ $\frac{4}{5} \times 100\% = 80\%$

b) $\frac{1}{5}$ _____

c) $\frac{9}{10}$ _____

d) $1\frac{1}{2}$ _____

e) $\frac{7}{10}$ _____

f) $3\frac{3}{4}$ _____

g) $15\frac{1}{4}$ _____

h) $\frac{3}{20}$ _____

i) $\frac{17}{25}$ _____

j) $\frac{1}{3}$ _____

k) $\frac{7}{50}$ _____

l) $\frac{1}{2}$ _____

m) $\frac{16}{20}$ _____

n) $\frac{1}{4}$ _____

Answers to Exercise Six

b) 20 %	c) 90 %	d) 150 %	e) 122.0 %
f) 375 %	g) 1 525 %	h) 15 %	i) 68 %
j) $33\frac{1}{3}$ %	k) 14 %	l) 50 %	m) 80 %
n) 25 %			

Method Two:

To change a common fraction to an equivalent percent, first write the common fraction as a decimal. Then multiply the decimal by 100% (move the decimal point two places to the **right**).

$$\frac{3}{8} = \underline{\quad} \%$$

$$
\begin{array}{r}
0.375 \\
8\overline{)3.000} \\
2\,4 \\
\hline
60 \\
56 \\
\hline
40
\end{array}
$$

$$\frac{3}{8} = 0.375 \qquad 0.37\,5 = 3.75$$

$$\frac{1}{3} = \underline{\quad} \%$$

$$
\begin{array}{r}
0.33\overline{3} \\
3\overline{)1.000}
\end{array}
$$

$$0.33\,.\overline{3} = 33.\overline{3}\%$$

also written as $33\frac{1}{3}\%$

$$\frac{11}{12} = \underline{\quad} \%$$

$$
\begin{array}{r}
0.9166 \\
12\overline{)11.000} \\
10\,8 \\
\hline
20 \\
12 \\
\hline
80 \\
72 \\
\hline
8
\end{array}
$$

$$0.91\,.\overline{6} = 91.\overline{6}\%$$

Exercise Seven

Change each common fraction to an equivalent percent by first writing a decimal.

a) $\dfrac{1}{12} = 0.08\overline{3}$ $0.08\overline{3} \times 100\% = 8.\overline{3}\%$ b) $\dfrac{1}{8}$ _____

c) $\dfrac{5}{8}$ _____

d) $\dfrac{7}{8}$ _____

e) $12\dfrac{1}{5}$ _____

f) $\dfrac{2}{3}$ _____

g) $7\dfrac{1}{3}$ _____

h) $\dfrac{5}{16}$ _____

i) $\dfrac{5}{6}$ _____

j) $\dfrac{4}{9}$ _____

k) $\dfrac{1}{6}$ _____

l) $\dfrac{40}{45}$ _____

Answers to Exercise Seven

b) 12.5 %	c) 62.5 %	d) 87.5 %	e) 1 220 %
f) 66.6 %	g) 733.3 %	h) 31.25 %	i) 83.3 %
j) 44.4 %	k) 16.6 %	l) 88.8 %	

➡ The method you use to change a common fraction to a percent will depend on the numbers you are working with. Choose whichever method seems easier for the situation. You will also memorize many equivalencies as you work with them. But you should definitely memorize $\dfrac{1}{3} = 33\dfrac{1}{3}\%$ and $\dfrac{2}{3} = 66\dfrac{2}{3}\%$.

Changing Percents to Common Fractions

You know that percents are a form of fraction with an unwritten denominator of 100. A **%** sign is used.

> To change any percent to a decimal, common fraction, or mixed number, divide by 100 and remove the percent sign.

To change a percent to a

common fraction:

- write the numerals in the percent as the numerator.

- write 100 as the denominator. (Remember that the line in a fraction can be a divided by sign, so 58% = $\dfrac{58}{100}$ is the same as 58 ÷ 100.)

- remove the % sign.

- simplify the fraction: $\dfrac{58}{100} = \dfrac{29}{50}$

Examples: Write each percent as a common fraction.

$$38\% = \frac{38}{100}\left(\frac{\div 2}{\div 2}\right) = \frac{19}{50}$$

$$25\% = \frac{25}{100}\left(\frac{\div 25}{\div 25}\right) = \frac{1}{4}$$

$$3\% = \frac{3}{100}$$

Note that **percents greater than or equal to 100 become improper fractions** which will be rewritten as mixed numbers.

$$110\% = \frac{110}{100} = 1\frac{10}{100} = 1\frac{1}{10}$$

$$\begin{array}{r} 1 \\ 100\overline{)110} \\ \underline{100} \\ 10 \end{array}$$

$$120\% = \frac{120}{100} = 1\frac{20}{100} = 1\frac{1}{5}$$

$$\begin{array}{r} 1 \\ 100\overline{)120} \\ \underline{100} \\ 20 \end{array}$$

➡ Remember 100% is the whole thing. **100% = 1**

Exercise Eight

Change each percent to a common fraction. Simplify to lowest terms.

a)   31% = _____

b)   11% = _____

c)   2% = _____

d)   78% = _____

e)   20% = _____

f)   50% = _____

g)   75% = _____

h)   60% = _____

i)   8% = _____

j)   95% = _____

k)　16% = _____

l)　3% = _____

m)　74% = _____

n)　100% = _____

o)　135% = _____

p)　750% = _____

q)　400% = _____

r)　320% = _____

s)　180% = _____

t)　282% = _____

Answers to Exercise Eight

a) $\frac{31}{100}$

b) $\frac{11}{100}$

c) $\frac{1}{50}$

d) $\frac{39}{50}$

e) $\frac{1}{5}$

f) $\frac{1}{2}$

g) $\frac{3}{4}$

h) $\frac{3}{5}$

i) $\frac{2}{25}$

j) $\frac{19}{20}$

k) $\frac{4}{25}$

l) $\frac{3}{100}$

m) $\frac{37}{50}$

n) 1

o) $1\frac{7}{20}$

p) $7\frac{1}{2}$

q) 4

r) $3\frac{1}{5}$

s) $1\frac{4}{5}$

t) $2\frac{41}{50}$

Percents Less than 1%

Sometimes a percent **smaller than 1%** is used. For example, you will hear amounts such as $\frac{1}{4}$ % or $\frac{1}{8}$ % or $\frac{1}{2}$ % on the news about the Bank of Canada rate and the rise and fall of inflation. These are small amounts. Sometimes the expression " $\frac{1}{2}$ of a percentage point" is used instead of " $\frac{1}{2}$ % ".

What is $\frac{1}{4}$ %?

$\frac{1}{4}$ % is $\frac{1}{4}$ **of 1%**.

$$1\% = \frac{1}{100} \quad \text{so} \quad \frac{1}{4} \text{ of } 1\% = \frac{1}{4} \times \frac{1}{100} = \frac{1}{400}$$

$$\frac{1}{4}\% = 0.25\% = 0.0025$$

What is $\frac{1}{2}$ %?

$$\frac{1}{2}\% = \frac{1}{2} \text{ of } 1\% = \frac{1}{2} \times \frac{1}{100} = \frac{1}{200}$$

$$\frac{1}{2}\% = 0.5\% = 0.005$$

To work with percents less than 1%, change the percent to a decimal by dividing by 100 (move decimal point two places to the left).

$$0.2\% = 0.002 \qquad\qquad 0.75\% = 0.0075$$

If the percent is expressed as a common fraction, do this:

- Write the common fraction percent as a decimal percent.

- Divide by 100 (move decimal point two places left).

$$\frac{1}{2}\% = 0.5\% = 0.005 \qquad\qquad \frac{1}{4}\% = 0.25\% = 0.0025$$

$$\frac{1}{8}\% = 0.125\% = 0.00125$$

Exercise Nine

Change each percent to an equivalent decimal.

a) $\frac{1}{2}$ %   =_____

b)   0.6%   = _____

c) $\frac{3}{10}$ %   =_____

d) $\frac{3}{5}$ %   = _____

e)   0.75%   =_____

f) $\frac{3}{4}$ %   = _____

g)   0.5%   =_____

h) $\frac{1}{4}$ %   = _____

i)   0.125%   =_____

j) $\frac{1}{3}$ %   = _____

k) $\frac{1}{8}$ %   =_____

l) $\frac{5}{8}$ %   = _____

m) $\frac{1}{16}$ %   =_____

n) $\frac{2}{5}$ %   = _____

Answers to Exercise Nine

a) 0.005   b) 0.006   c) 0.003   d) 0.006
e) 0.0075   f) 0.0075   g) 0.005   h) 0.0025
i) 0.00125   j) 0.003   k) 0.00125   l) 0.00625
m) 0.000625   n) 0.004

$16\frac{2}{3}\%,\ 33\frac{1}{3}\%,\ 66\frac{2}{3}\%,\ 83\frac{1}{3}\%$ ...

These percents will become repeating decimals. For example

$$33\frac{1}{3}\% = 33.\overline{3}\% = 0.33\overline{3}$$

$$66\frac{2}{3}\% = 66.\overline{6}\% = 0.66\overline{6}$$

It is usually more convenient to use the common fraction equivalent of these percents.
Memorize them, or make a note on a special paper and post it near your work space.

$$33\frac{1}{3}\% = 33\frac{1}{3} \div 100 = \frac{100}{3} \times \frac{1}{100} = \frac{1}{3}$$

$$66\frac{2}{3}\% = 66\frac{2}{3} \div 100 = \frac{200}{3} \times \frac{1}{100} = \frac{2}{3}$$

$$16\frac{2}{3}\% = \frac{1}{6}$$

$$33\frac{1}{3}\% = \frac{1}{3}$$

$$66\frac{2}{3}\% = \frac{2}{3}$$

$$83\frac{1}{3}\% = \frac{5}{6}$$

Review of Equivalent Common Fractions, Decimals, and Percents

Complete this chart. These are equivalents that you will often use, so use this chart for reference. Memorize as many equivalents as you can. You may wish to put other equivalents on the chart.

Common Fraction	Decimal	Percent
$\dfrac{1}{4}$		
	0.5	
		75%
$\dfrac{1}{8}$		
	0.375	
		62.5%
$\dfrac{7}{8}$		
	0.2	
		40%
$\dfrac{3}{5}$		
	0.8	
$\dfrac{2}{2} = \dfrac{3}{3} = \dfrac{4}{4} = \dfrac{5}{5} = 1$		

$\frac{1}{6}$		
	$0.3\overline{3}$	
		$66.\overline{6}\%$
$\frac{5}{6}$		
	0.1	
		30%
$\frac{7}{8}$		
	0.9	
	1	

Answers to Review of Equivalent Common Fractions, Decimals, and Percents

Common Fraction	Decimal	Percent
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{1}{8}$	0.125	12.5%
$\frac{3}{8}$	0.375	37.5%
$\frac{5}{8}$	0.625	62.5%
$\frac{7}{8}$	0.875	87.5%
$\frac{1}{5}$	0.2	20%
$\frac{2}{5}$	0.4	40%
$\frac{3}{5}$	0.6	60%
$\frac{4}{5}$	0.8	80%
$\frac{2}{2}=\frac{3}{3}=\frac{4}{4}=\frac{5}{5}=1$	1	100%
$\frac{1}{6}$	$0.1\overline{6}$	$16.\overline{6}$ %
$\frac{1}{3}$	$0.3\overline{3}$	$33.\overline{3}$ % or $33\frac{1}{3}$ %
$\frac{2}{3}$	$0.6\overline{6}$	$66.\overline{6}$ % or $66\frac{2}{3}$ %
$\frac{5}{6}$	$0.8\overline{3}$	$83.\overline{3}$ %
$\frac{1}{10}$	0.1	10%
$\frac{3}{10}$	0.3	30%
$\frac{7}{8}$	0.875	87.5%
$\frac{9}{10}$	0.9	90%
e.g. $\frac{9}{9}$	1	100%

Topic A: Self-Test

Mark   /26   Aim 22/26

A. Write these percents in numeral form.   **3 marks**

   a) sixty-two and one-half percent   _____

   b) eight percent   _____

   c) one hundred six and one-fifth percent   _____

B. Write these percents in words.   **3 marks**

   a) 72% _____

   b) $120\frac{1}{4}$ % _____

   c) $\frac{3}{4}$ % _____

C. Change the percents to decimal fractions.   **10 marks**

   a)   32%   _____   b)   10% _____

   c)   18.5%   _____   d)   125% _____

   e)   1%   _____   f)   0.25% _____

   g) $44\frac{1}{2}$ %   _____   h) $\frac{3}{4}$ % _____

   i) $\frac{4}{5}$ %   _____   j) $\frac{2}{3}$ % _____

D. Change these percents to common fractions in lowest terms.   **6 marks**

    a) 16%   _____   b) 20%   _____

    c) 106%   _____   d) 100%   _____

    e) 75%   _____   f) 5%   _____

E. Change these common fractions to equivalent percents.   **4 marks**

    a) $\frac{4}{5}$ _____   b) $\frac{1}{4}$ _____

    c) $\frac{3}{8}$ _____   d) $\frac{11}{12}$ _____

Answers to Topic A Self-Test

Part A

a) 62.5 % **or** $62\frac{1}{2}$ %   b) 8 %   c) 106.2 % **or** $106\frac{1}{5}$ %

Part B

a) seventy-two percent   b) one hundred twenty and one-quarter percent
c) three-quarters percent

Part C

a) 0.32	b) 0.10	c) 0.185	d) 1.25
e) 0.01	f) 0.0025	g) 0.445	h) 0.0075
i) 0.008	j) $0.00\overline{6}$		

Part D

a) $\frac{4}{25}$	b) $\frac{1}{5}$	c) $1\frac{3}{50}$	d) 1
e) $\frac{3}{4}$	f) $\frac{1}{20}$		

Part E

a) 80 %	b) 25 %	c) 37.5 %	d) $91.\overline{6}$ %

Unit 2 Review

1. Change these decimals to percents.

a) 0.75 _____   b) 0.34 _____

c) 0.156 _____   d) 0.03 _____

e) 0.0035 _____   f) 0.625 _____

g) 0.048 _____   h) 0.67 _____

i) 3.45 _____   j) 2.37 _____

2. Change each percent to a decimal.

a) 59% _____   b) 42% _____

c) 39.5% _____   d) 152% _____

e) 4.3% _____   f) 1% _____

g) $3\frac{1}{2}\%$ _____   h) $5\frac{3}{4}\%$ _____

i) $32\frac{1}{3}\%$ _____   j) $4\frac{1}{4}$ _____

k) $1\frac{4}{5}\%$ _____   l) $4\frac{3}{5}\%$ _____

3. Change each fraction to an equivalent percent.

a) $\frac{6}{10}$ _____   b) $1\frac{3}{4}$ _____

c) $\frac{1}{5}$ _____   d) $4\frac{3}{20}$ _____

e) $\dfrac{5}{19}$ _____ f) $\dfrac{9}{20}$ _____

g) $1\dfrac{1}{4}$ _____ h) $2\dfrac{1}{3}$ _____

i) $\dfrac{7}{50}$ _____ j) $3\dfrac{17}{25}$ _____

4. Change each percent to a common fraction. Simplify your answer.

a) 16% _____ b) 25% _____

c) 110% _____ d) 95% _____

e) 650% _____ f) 284% _____

g) $\dfrac{1}{4}$ % _____ h) $\dfrac{1}{2}$ % _____

i) $\dfrac{1}{8}$ % _____ j) $\dfrac{3}{5}$ % _____

k) 0.125% _____ l) $\dfrac{1}{16}$ % _____

m) $\dfrac{5}{8}$ % _____ n) 0.7% _____

o) $\dfrac{3}{4}$ % _____ p) 0.25% _____

5. Change each percent to a common fraction. These few should be memorized.

a) $33\frac{1}{3}$ %   _____

b) $66\frac{2}{3}$ %   _____

c) $16\frac{2}{3}$ %   _____

d) $83\frac{1}{3}$ %   _____

TEST TIME!

Ask your instructor for
the Practice Test for this unit.

Once you've done the practice test,
you need to do the unit 2 test.

Again, ask your instructor for this.

Good luck!

Unit 3

Working with Percent

Working with Percent

In this unit you will learn to solve three types of percent problems:

- Finding a given percent of a number (**finding the part**).

- Finding what percent one number is of another number (**finding the %**).

- Finding a number when a percent of it is given (**finding the whole**).

Each type of percent problem can be solved using the following proportion:

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

Both ratios in this proportion use the same order of comparison because in the ratio $\dfrac{\%}{100}$ the % represents a part (is) and 100 is the whole (of).

Percent problems involve knowing three pieces of information:

1) the part (the "is" part)
2) the whole (the "of" part)
3) the percent

You will be given two pieces of information and you will find the third. That is, the problems will give two terms of the proportion and you will solve for the missing term. Because these are problems of percent, the **100** is always known to you and will always be in the same position in the proportion.

Remember **Using Cross Multiplication to Solve a Proportion:**

$$\frac{N}{4} = \frac{6}{8} \longrightarrow 4 \times 6 = 8 \times N \longrightarrow 24 = 8N \longrightarrow \frac{24}{8} = \frac{8N}{8} \longrightarrow 3 = N$$

Topic A: Finding a Percent of a Number

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

In problems in which you find a percent of a number, the **missing term** is the **part (is)**. You will be given the % and the whole (of), and you know the 100.

Example A:

What is 25% of 40?

Part (is)   = N

Whole (of) =   40

%   =   25

$$\frac{is}{of} = \frac{\%}{100} \longrightarrow \frac{N}{40} = \frac{25}{100}$$

Solve the proportion.

Simplify if possible

$$\frac{N}{40} = \frac{\cancel{25}^{1}}{\cancel{100}_{4}}$$

Cross multiply

$N \times 4 \; = 40 \times 1$

$4N \quad = 40$

Divide

$N \quad = 40 \div 4 = 10$

25% of 40 = 10

Example B: What is 20% of 18?

part (is)   = N

whole (of) = 18

%   = 20

$$\frac{is}{of} = \frac{\%}{100} \longrightarrow \frac{N}{18} = \frac{20}{100}$$

Solve the proportion:

$$\frac{N}{18} = \frac{\cancel{20}^1}{\cancel{100}_5}$$

$$5N = 18$$

$$N = 18 \div 5 \qquad = 3\frac{3}{5} \qquad 20\% \text{ of } 18 = 3\frac{3}{5}$$

The following examples all ask you to find a percent of a number. The missing term is the **part (the "is" part)**. Look at the examples carefully so you'll recognize the wording.

- What is 14% of 60? $\qquad \dfrac{is}{of} = \dfrac{\%}{100} \qquad \longrightarrow \qquad \dfrac{N}{60} = \dfrac{14}{100}$

- Find 10% of 27. $\qquad \dfrac{is}{of} = \dfrac{\%}{100} \qquad \longrightarrow \qquad \dfrac{P}{27} = \dfrac{10}{100}$

- 5% of 15 is _____. $\qquad \dfrac{\%}{100} = \dfrac{is}{of} \qquad \longrightarrow \qquad \dfrac{5}{100} = \dfrac{X}{15}$

- 75% of 12 = _____. $\qquad \dfrac{\%}{100} = \dfrac{is}{of} \qquad \longrightarrow \qquad \dfrac{75}{100} = \dfrac{K}{12}$

➡ In percent problems, the number after the word **of** usually represents the **whole**.

Exercise One

Solve each problem by setting up the proportion.

$$\frac{is \text{ (part)}}{of \text{ (whole)}} = \frac{\%}{100}.$$

a) 20% of 18 = _____

b) 19% of 200 = _____

c)   25% of 44 = _____   d)   6% of 110 = _____

e)   3% of 33 = _____   f)   30% of 64 = _____

g)   50% of 60 is? _____   h)   30% of 40 is? _____

i)   What is 72% of $425? _____   j)   What is 20% of 85? _____

k)   3% of $72? _____   l)   What is 12% of 84? _____

m) What is 25% of $64? ____   n) 75% of 120 =?   _____

Answers to Exercise One

a) $3\frac{3}{5}$ or 3.6   b) 38   c) 11   d) $6\frac{3}{5}$ **or** 6.6

e) $\frac{99}{100}$ or 0.99   f) $19\frac{1}{5}$ or 19.2   g) 30   h) 12

i) $306   j) 17   k) $2.16   l) $10\frac{2}{25}$ **or** 10.08

m) 16   n) 90

Percents Greater Than or Equal to 100%

Remember that 100% = 1.

100% of anything is the whole thing. If you spend 100% of your pay cheque, you spend the whole thing. If you get 100% on a test, you have the whole thing correct.

If you have **more than 100%** you have **more than the whole thing**. If you spend 110% of your paycheque, you spent more than you earned, and you may be in trouble! It is hard to get more than 100% on a test unless the instructor has given bonus marks for extra questions. You may hear of percents more than 100% in increases, such as costs of housing or inflation. For example, "The Browns just sold their house and made a 200% profit." This means they got back what they paid and two times more!

If a percent is less than (<) 100, it is less than the whole thing.

90% of 50 = 45

If a percent is 100, it equals the whole thing.

100% of 50 = 50

If a percent is more than (>) 100, it is more than the whole thing.

120% of 50 = 60

Exercise Two

Look at the percent. Is it < 100, = 100, or > 100? Circle the correct answer for each question. <u>Do not solve the problems.</u>

a) 200% of 10 is

    i)   equal to 10

    ii) less than 10

    iii) greater than 10

b) 50% of 0.25 is

    i)   equal to 0.25

    ii) less than 0.25

    iii) greater than 0.25

c) 90% of 75 is

    i)   equal to 75

    ii) less than 75

    iii) greater than 75

d) $33\frac{1}{3}$% of 15 is

    i)   equal to 15

    ii) less than 15

    iii) greater than 15

e) 100% of 100 is

    i)   equal to 100

    ii) less than 100

    iii) greater than 100

f) 127% of 936 is

    i)   equal to 936

    ii) less than 936

    iii) greater than 936

Answers to Exercise Two

a) iii) greater than 10   b) ii) less than 0.25   c) ii) less than 75
d) ii) less than 15   e) i) equal to 100   f) iii) greater than 936

Exercise Three

Use the proportion method to solve these questions.

a) $16\frac{2}{3}$ % of 12 = _____

b) What is 60% of 15? _____

c) 75% of 144 is? _____

d) 30% of 90 = _____

e) What is $37\frac{1}{2}$ % of 80? _____

f) 25% of 52 is? _____

g) Find 8.2% of 300. _____

h) 260% of 45 is? _____

i) What is 109% of 200? _____

j) 98.75% of 50 = _____

k) Find 2% of 720. _____

l) 5% of 180 is? _____

m) 90% of 700 = _____

n) What is 85% of 600? _____

o) Find 46% of 90. _____

p) What is 12.5% of 90? _____

q) $37\frac{1}{2}$ % of 80 is? _____

r) 115% of 250 = _____

s) 2.5% of 300 = _____

t) Find $33\frac{1}{3}$ % of 252. _____

u) What is $66\frac{2}{3}$ % of 324? _____

v) $3\frac{1}{4}$ % of 200 is? _____

w) $62\frac{1}{2}$ % of 872 = _____

x) Find $16\frac{2}{3}$ % of 264. _____

Answers to Exercise Three

a) 2	b) 9	c) 108	d) 27
e) 30	f) 13	g) 24.6	h) 117
i) 218	j) 49.375	k) 14.4	l) 9
m) 630	n) 510	o) 41.4	p) 11.25
q) 30	r) 287.5	s) 7.5	t) 84
u) 216	v) 6.5	w) 545	x) 44

Taxes

The amount of tax to be paid is calculated by finding a percent of a number. The tax rate is usually given as a percent. The basic proportion for these problems is:

$$\frac{\text{tax (part)}}{\text{taxable amount (whole)}} = \frac{\% \text{ tax}}{100}$$

Please note that the tax rates used in the questions in this book are for the year 2010 and are subject to change.

The British Columbia Harmonized Sales Tax (HST) is 12%. In B.C., the provincial portion of the harmonized sales tax does not have to be paid on children's clothes, food, books, gasoline and diesel fuel and other special items.

Example A: How much HST (12%) will be charged on a new kitchen table that cost $125?

use proportion:

$$\frac{H.S.T}{\$125} = \frac{12}{100}$$

100 HST = $1500

HST = $875 ÷ 100 = $15.00

HST on a $125 table is $15.00.

Exercise Four

Find the total cost of each item. All are to be taxed with HST.

	Purchase Price	HST 15%	Total Cost
a)	clothes $130		
b)	washing machine $589		
c)	compact disc $18.99		
d)	garden tools $48.50		
e)	lumber $250		
f)	new car $10 000		
g)	shoes $59.99		
h)	television $489		

	Purchase Price	HST 15%	Total Cost
a)	clothes $130	$15.60	$145.60
b)	washing machine $589	$70.68	$659.68
c)	compact disc $18.99	$2.28	$21.27
d)	garden tools $48.50	$5.82	$54.32
e)	lumber $250	$30.00	$280.00
f)	new car $10 000	$1200	$11 200
g)	shoes $59.99	$7.20	$67.19
h)	television $489	$58.68	$547.68

Income Tax is charged at different percentages according to the amount of a person's taxable income. The first $28 000 of taxable income is taxed at 17%. Note that other tax rules and charges may apply in real situations.

Example A: If a person's taxable income for the year is $23 400, what amount of income tax will that person pay?

To use the proportion method, do this:

$$\frac{tax}{income} \rightarrow \frac{T}{\$23\ 400} = \frac{17}{100}$$

$100T = 17 \times \$23\ 400$

$100T = 397\ 800$

$T = \$397\ 800 \div 100 = \$3\ 978$

The income tax on $23 400 is $3 978.

Exercise Five

Calculate the income tax for the annual taxable earnings listed. These amounts are all under $28 000, so the tax rate is 17%.

a) $18 500   _____

b) $27 620   _____

c) $15 365   _____

d) $25 900   _____

Answers to Exercise Five

a) $3 145   b) $4 695.40   c) $2 612.05   d) $4 403

If you are interested in the tax rates and the basic calculation of income tax for taxable incomes higher than $28 000, talk to your instructor. Your instructor will help you to find out various rates and will show you how to do the calculations.

Cross-Border Shopping

The Canadian (CAN) and American (US) dollars are not equal in value. The **exchange rate** (the value of one Canadian dollar compared to a dollar from another country) changes often; the current rate is usually available from banks, on the news, in the newspapers and on a web site. In the winter of 2010, the Canadian dollar was around $0.92 of an American dollar (ratio is $1.00 CAN:$0.92 US), so CAN money was valued at 92% of US money.

⇨ To find the value of one US dollar in Canadian funds, use this proportion:

$$\frac{\$1 \text{ CAN}}{\$0.92 \text{ US}} = \frac{\text{N CAN}}{\$1 \text{ US}}$$

$N = \$1 \div \$0.92 = \$1.08$, so US money was valued at 108% of CAN money.

Note that the proportion changes as the exchange rate changes.

What if you buy in the United States?

- Change the US cost to the Canadian equivalent (multiply by 108% [varies]).

- If you have more than the purchases allowed (call the **Canada Border Service Agency** for information), the Canadian Customs charge **duty** on the Canadian value of your purchases. The percent of the duty (the rate) varies according to what the item is, where it was made and the duty rates of the day. For example, duty on poultry is 12.5%, on non-US cotton 25%, and on liquor 110%!

  Duty is gradually being eliminated under the Canada-US Free Trade Agreement. If an item is made in North America, there is no duty charged because of NAFTA (North American Free Trade Agreement)

- HST (12%) is charged on the duty and on the Canadian value of the purchases (which includes any US sales taxes).

Look at this example (assume $1.00 CAN = $0.92 US).

Men's leather shoes, US price   **$64.80**

US sales tax 6% $\left(\dfrac{N}{64.80} = \dfrac{6}{100}\right)$ 3.89

US total cost   68.69

Equivalent cost in CAN funds (US$ $\times$ 108%)

$\left(\dfrac{N}{68.69} = \dfrac{108}{100}\right)$ $74.19

Duty on leather shoes is 22.8%

$\left(\dfrac{N}{74.19} = \dfrac{22.8}{100}\right)$ $16.92

HST (12%) on CAN value plus duty

12% of ($74.19 + 16.92) is $\left(\dfrac{N}{91.11} = \dfrac{12}{100}\right)$ $10.93

The total cost of a pair of leather shoes priced at $64.80 in the United States will be the American price in Canadian funds + duty + HST = **$102.04 CAN**

Exercise Six

For each item, do the calculations using the duty and tax rates given. Assume $1.00 US is $1.08 CAN.

a) Groceries, US price $**75.**

   i) 3% US sales tax

   ii) US total cost

   iii) Equivalent value in CAN funds

   iv) Duty at 10.2% (No HST on food)

   v) Total cost in Canadian funds

b)   Cotton Clothing, made in Romania, US price **$98.**

   i) 5% US sales tax

   ii) US total cost

   iii) Equivalent value in CAN $

   iv) Duty at 25%

   v)   HST at 12% (on Canadian Value + Duty)

   vi) Total cost in Canadian funds

Answers to Exercise Six

a) i) $2.25   ii) $77.25   iii) $83.43   iv) $8.51   v) $91.94
b) i) $4.90   ii) $102.90   iii) $111.13   iv) $27.78   v) $16.67   vi) $155.58

Increases and Decreases, Discounts and Mark-ups

Increases (amount changing to more) and decreases (amount changing to less) are often given as a percent. For example,

- The Insurance Corporation of B.C. increased car insurance rates by 3.3% in March 2007.

- The number of acute care beds at the local hospital has decreased by 28% in the last year.

- The new work contract provides a 4% wage increase in the first year and a $2\frac{1}{2}$% increase in the second year.

The amount of an increase or decrease is calculated by **finding a percent of a number**. When the percent of an increase or decrease is given, the proportion is:

$$\frac{\text{amount of increase or decrease}}{\text{whole amount}} = \frac{\text{increase or decrease \%}}{100}$$

Discounts are a form of decrease. The discount is the amount taken off a price; it is the price reduction.

Sale prices (discounted prices) may be advertised as

"All items 20% off."
"Everything in stock reduced by 25% to 50%."
"33 $\frac{1}{3}$% savings!"
"2% discount for cash."

Decrease and discount problems may need to be solved in several steps. Sometimes the problems ask for:

- the amount of the decrease (may be called the "saving") (1 step).

- the amount left **after** the decrease (2 steps).

Example A: The sign says, "All winter coats 40% off." How much money will you **save** on a coat originally priced at $128.99?

    One step problem

    use proportion $\dfrac{decrease}{original\ price}$ $\rightarrow$ $\dfrac{D}{\$128.99} = \dfrac{40}{100}$

    $100D = 40 \times \$128.99$

    $D = \$5\ 159.60 \div 100 = \$51.596$ round to nearest cent $= \$51.60$

    You will save $51.60

Example B: The couch and chair are advertised in a $33\dfrac{1}{3}$ % price reduction sale. How much will you **pay** for a couch and chair originally priced at $798?

    Two step problem

    First: Find the amount of savings (the decrease).

        $\dfrac{saving}{full\ cost}$ $\rightarrow$ $\dfrac{S}{798} = \dfrac{33\dfrac{1}{3}}{100}$

    Second: **Subtract** the savings from the original amount.
                    original amount - savings = sale price

Step 1   Find the amount of savings.

$$\frac{S}{798} = \frac{33\frac{1}{3}}{100} \longrightarrow 798 \times 33\frac{1}{3} = S100 \longrightarrow \frac{798}{1} \times \frac{100}{3} = S100$$

$$26600 = S100 \longrightarrow \frac{26600}{100} = \frac{S\cancel{100}}{\cancel{100}} \longrightarrow S = \$266$$

Step 2   Find the sale price.

original amount - savings (decrease) = sale price

$798 - $266 = $532

The couch and chair will cost $532 on sale (plus HST of course, but you do not have to calculate tax for this problem).

Exercise Seven   Solve these problems. Round all answers to the nearest cent.

a) The employees agreed to take a 5% pay cut (reduction) so no-one will be laid off. If the pay rate was $15.50 per hour, how much less per hour will the workers earn?

b) "All shoes 25% off," says the sign. What will be the sale price of a pair of dress shoes originally priced at $69.98?

c) The work force at the factory has to be reduced by $16\frac{2}{3}$ % over the next two years. Early retirements, attrition (not replacing people who leave) and some lay-offs will be used. The work force is 3000 people right now. What is the planned size of the work force in two years?

d) 28% of the acute care beds in the hospital are going to be closed. The hospital has 175 acute care beds now.

    i)   How many beds will be closed?

    ii)   How many beds will the hospital have open for use after the bed closure?

Answers to Exercise Seven

a) $0.78   b) $52.48   c) 2 500 people
d) i) 49 beds   ii) 126 beds

Increases and mark-ups are calculated in the same way as decreases and discounts. However, an increase or mark-up is **added** to the original amount.

Example A: The auto insurance rate increased 19%. The basic insurance rate for Don's car was $550 before the increase. What is the basic insurance after the increase?

Step 1 Calculate the amount of the increase

$$\frac{amount\ of\ increase}{present\ \cos t} \quad \rightarrow \quad \frac{N}{550} = \frac{19}{100}$$

$$N\,100 = 550 \times 19 \quad \longrightarrow \quad N = 104.5$$

Step 2 **Add** the amount of increase to the original amount.

original amount + increase = new insurance cost

$550 + $104.50 = $654.50

Don's new basic insurance is $654.50.

Mark-ups are the amount added to the cost price before an item is resold. Many factors must be considered when businesses decide on the percent of the mark-up:

- all costs of operating a business

- the profit wanted

- the community the business is in

- the competition the business has

For example, the mark-up on leather shoes may be 45%, but on running shoes it may be 60%. Kitchen appliances might have a 42% mark-up, while lawn mowers might have a 55% mark-up.

Example A:

A shoe seller pays $40.00 per pair of running shoes from the factory. The shoe seller makes the mark up 75%. What is the selling price of the shoes?

$$\frac{mark\ up\ cost}{original\ cost} \rightarrow \frac{N}{40} = \frac{75}{100} \longrightarrow 3000 = N100 \rightarrow N = 30$$

Add the mark up to the original cost to get the selling price of the shoes:

$75 + \$30 = \$105.00$

Wage Increase

Having a wage increase at work is always a good thing! Often the raise will be given as a percentage. That means that everyone will see more money on their pay cheque, but they will each have a different amount because they all get paid a different amount to start with.

Example B:

The boss at A-1 House Painting will give a 1.5% wage increase to the 10 employees.
a) 3 staff are paid the minimum wage of $8.00 an hour.
b) the other 7 staff are paid $12.00 an hour.

a) $\dfrac{increase}{present\ wage} \rightarrow \dfrac{I}{8} = \dfrac{1.5}{100} \longrightarrow \dfrac{increase}{present\ wage} \rightarrow \dfrac{I}{8} = \dfrac{1.5}{100}$

$8 \times 1.5 = I100 \longrightarrow I = \$0.12$

The new wage will be the old wage plus the increase:
$8.00 + \$0.12 = \$8.12$ per hour

b) $\dfrac{increase}{present\ wage} \rightarrow \dfrac{I}{12} = \dfrac{1.5}{100} \longrightarrow 12 \times 1.5 = I100 \longrightarrow I = \$0.18$

The new wage will be the old wage plus the increase:

$12.00 + \$0.18 = \$12.18$ per hour

Exercise Eight

Solve the problems. Round money to the nearest cent.

a) If the mark-up on the craft supplies was set at 75%, calculate the selling price for these items and complete the chart.

Cost Price to Business	Mark-up (75%)	Selling Price
silk flowers $1.48	$\dfrac{N}{1.48} = \dfrac{75}{100}$ = $1.11	$1.48+$1.11=$2.59
styrofoam cones $0.40		
lace $1.40 metre		
stuffing $4.50 bag		
beads $3.20 dozen		

b) The population of the town has increased 30% since the pulp mill was built.
The population before the pulp mill was 8 436 people. What is the population now?
(Round to the nearest person.)

c) The wage contract gave the workers a 4% increase in the first year and a $2\frac{1}{2}$% increase in the second year. If the hourly rate of pay was \$12.45 before the new contract, calculate the following:

    i)   The amount of the increase per hour in the first year.

    ii)   The hourly pay rate in the first year. (old pay + increase = first year rate)

    iii) The amount of pay increase in the second year. (Note—use the new hourly pay rate from the first year to calculate the increase for the second year.)

    iv) The hourly pay rate in the second year. (first year rate + increase = second year rate)

d) Everyone in the apartment building received a notice that the rent would be increased 8% on September 1.

i)   If the one bedroom apartment rents for $375 a month, how much will the increase be?

ii)   What is the rent for the one bedroom apartment after September 1?

iii)   The two bedroom apartment is $425 a month. How much is the increase per month?

iv)   What is the rent for a two bedroom apartment after September 1?

v)   If you rent a two bedroom apartment, how much more will you have to pay per year **after** the increase than you paid per year before the increase?

e) Joao's wage at his new job was $15 an hour. He loves his job <u>and</u> his boss promised him an increase of pay of 10% after his first 500 hours of work. What will his new pay be?

Answers to Exercise Eight

Cost Price to Business	Mark-up (75%)	Selling Price
silk flowers $1.48	*$1.11*	*$1.48+$1.11=$2.59*
styrofoam cones $.40	$0.30	$0.70
lace $1.40 metre	$1.05	$2.45
stuffing $4.50 bag	$3.38	$7.88
beads $3.20 dozen	$2.40	$5.60

b) 10 967 people   c) i) $0.50   c) ii) $12.95   c) iii) $0.32

c) iv) $13.27   d) i) $30   d) ii) $405   d) iii) $34

d) iv) $459   d) v) $34   e) $16.50

Commission and Tips

Salespeople may receive a **commission** as part or all of their pay. The business owner pays the salesperson an agreed-upon percent of the selling price of the product.

- Real estate agents are paid by commission on their sales.

- Car and truck salespeople may be paid a small salary per month, but their main income is the commission on the vehicles they sell.

Tips are appreciation payments for service. The customer gives tips directly to the worker. Taxi drivers, waiters, bellhops and chambermaids in hotels often receive a minimal hourly wage. A large part of their earnings is from **tips**. In restaurants, expect to leave at least a 15% tip for adequate service.

To calculate the amount of a commission (or a tip), find the percentage of the total amount using the proportion

$$\frac{\text{commission (part)}}{\text{total amount (whole)}} = \frac{\text{commission \%}}{100}$$

Commission problems often have several steps. You may have to

- add together several items to find the total of sales.

- subtract a base amount for which salespeople do not receive a commission.

- add the amount of commission to the basic wage to find out how much the person earned.

Example A:

The bill for the excellent dinner at the restaurant was $56.40. The service had been good and the waiter very pleasant so Bill and Diane wanted to leave at least a 15% tip.

i) How much is the tip?

$$\frac{tip}{bill} \rightarrow \frac{N}{56.40} = \frac{15}{100} \longrightarrow 56.40 \times 15 = N100 \longrightarrow N = \$8.46$$

Bill will round this amount to $8.50

ii) How much will he pay for the meal and the tip?

cost of dinner $56.40
$\underline{+ \text{tip} \quad 8.50}$
$64.90

In a **real** situation, we would probably round the amount of the bill to the nearest dollar and then calculate the tip.

Example B:

The salespeople at XW Ford receive a monthly salary of $1 000. They also receive a 12% commission on any sales over $35 000 in a month. This means they are expected to sell $35 000 worth of vehicles every month to earn the $1000 salary. If a saleswoman made $54 000 in sales one month, what would her gross earning be?

You are asked to find the gross monthly earnings. What do you know?
        - She earns $1 000 per month.
        - She earns 12% commission on sales over $35 000.
        - She had $54 000 in sales.

Step 1   Subtract the base amount for which she will not earn a commission from her total sales.

$$\text{total sales - base amount = amount of sales that commission}$$
$$\text{will be paid for}$$

$54 000 - $35 000 = $19 000$ in commissionable sales

Step 2   Calculate the commission.

$$\frac{commission}{commissionable\ sales} \quad \rightarrow \quad \frac{X}{19000} = \frac{12}{100}$$

$19\ 000 \times 12 = X100 \quad \rightarrow \quad 228\ 000 = X\ 100 \quad \rightarrow \quad X = \$2\ 280$

Step 3   Add the salary and commission to find gross earnings

$1000 + $ 2 280 = $3 280.

The saleswoman earned $3 280.

Exercise Nine

Solve these problems.

a) A real estate agent sells a home for $128 500. How much is the commission at 7% of the selling price? (Note that some of the commission goes to the real estate company and some to the salesperson.)

b) A clerk sold $18 000 worth of clothes last year. He was paid a 15% commission. How much was his commission?

c) Mr. Green receives a weekly salary of $325 plus a commission of 10% on all sales he makes over $1 500. Last week Mr. Green sold $3 500 worth of merchandise. How much money did he earn last week?

d) George sells used cars. He gets a 9% commission on all his sales. If he sold a total of 5 cars priced at $2 295, $3 450, $8 600, $1 049, and $5 299 last month, how much money did he earn?

e) The final bill at the restaurant is $160 and you want to leave a 15% tip.

   i)   What amount of tip should you leave?

   ii)   What is the total cost (bill and tip)?

   iii) The total cost is to be shared amongst eight people. How much will each person pay?

More Problems for Finding a Percent of a Number

a) Joe was having trouble at school because he was absent 40% of the last 35 days of school. How many days did he miss?

b) To successfully pass the course, the student must get at least 80% on the test. The test is out of 125. What mark will give the student 80%?

c) The Masons invited 275 people to their daughter's wedding. They were happy when 90% of the people invited were able to attend. How many people came to the wedding? (round to the nearest person)

d) George made a $12\frac{1}{2}$% down payment on a new car which cost $3 200. How much was the down payment?

e) In B.C., employers are required to pay 6% holiday pay to all employees. Holiday pay is added to regular salary if a paid vacation is not taken. The young grocery clerk who earns $8.00 an hour worked 25 hours last week.

i)   What amount of holiday pay is he eligible for?

ii)   His employer pays holiday pay on each cheque to part-time employees. What are his total earnings for the week? (salary + holiday pay)

f) Nutrition experts recommend that no more than 30% of the food calories a person consumes should be from fats. Foods such as fatty meat, dairy products with lots of butter fat, cooking oils, margarine, some salad dressings, and nuts contain a high percentage of fat. If a person's daily intake of calories is 2 560, what is the most number of calories that should be from fat?

g) 40% to 60% of a person's body weight is water. Good health depends on keeping the body's fluid (water) level balanced. Excessive sweating, illness such as diarrhea and some "crash" diets can upset this fluid balance.

    i)   Dave is a distance runner who weighs 70.5 kg. If his body weight is 60% water, what is the weight of the water contained throughout his body?

    ii)   Jack weighs 103 kg. He has a lot of body fat which contains less water, so only about 43% of his body weight is water. What is the weight of the water contained in Jack's body?

Answers to More Problems for Finding a Percent of a Number

a) 14 days   b) 100 marks   c) 248 people   d) $400
e) i) $12.00   e) ii) $212.00   f) 768 calories   g) i) 42.3 kg
g) ii) 44.29 kg

Topic A: Self-Test

Mark /18   **Aim** 15/18

A. Answer the following.   **7 marks**

a)   6% of 30 =   _____

b) $\frac{1}{4}$ % of 48 is?   _____

c)   What is 75% of 200?   _____

d) $12\frac{1}{2}$ % of 8 =   _____

e)   Find 131% of 400.   _____

f)   1.5% of 21 is?   _____

g)   Write the general proportion that can be used to solve percent problems.

_____

B. Solve the problems. (2 marks each, except c)   **11 marks**

a) The $125 000 house is insured for 85% of its value against fire damage. How much money should the owner receive if the house is destroyed by fire?

b) Charlotte sells leisure clothes in her small town for a large national company. She receives $500 a month, and a 20% commission on all monthly sales over $1 000. What are her monthly earnings if her total sales are $2300?

c) The barbeque was originally priced at $599 but Jack bought it during a 35% off, end-of-season sale.

    i)   What was the sale price? (1 mark)

    ii)   Calculate the harmonized sales tax (12%). (1 mark)

    iv) Give the total cost of Jack's barbeque. (1 mark)

d) The contract to deliver phone books was split between two people. Jeff was given 60% of the books to deliver. The total number of books was 3 600. How many books did Jeff have to deliver?

e) This test is scored out of 19. On self-tests you should aim to get 80% to 85%. On this test your aim is 84%. Calculate, to the nearest whole number, the aim for this test.

Answers to Topic A Self-Test

Part A

a) 1.8	b) 0.12	c) 150	d) 1
e) 524	f) 0.315	g) $\dfrac{part}{whole} = \dfrac{\%}{100}$ or $\dfrac{is}{of} = \dfrac{\%}{100}$	

Part B

a) $106250	b) $760.00	c) i) $389.35	c) ii) $46.72
c) iii) $436.07	d) 2 160 books	e) 16 marks	

Unit 3 Review

1) Solve each problem by setting up a proportion

   a) 50% of 22

                                                               b) 25% of 36

   c) 150% of 7

                                                               d) 12.5% of 48

   e) 20% of 16

                                                               f) 18% of 40

   g) 30% of 4

                                                               h) 5% of 45

   i) what is 13% of 50?

                                                                j) 85% of \$165.00

   k) $\frac{1}{4}$ % of 20

                                                              l) 0.5% of 122

   m) 2% of \$7 800.00

                                                              n) 3.5% of 60

2) Use the proportion method to solve these questions

a) 75% of 12

b) $3\frac{1}{4}$% of 200 is?

c) what is 16% of 34?

d) 90% of 75 is

e) what is 14% of 60?

f) find 10% of 27

g) 5% of 15 is

h) $16\frac{2}{3}$% of 163

3) Find the total cost of each item, using 12% HST

	Item	Cost	HST (12%)	Total Cost
a)	t-shirt	$14.99	$\dfrac{X}{14.99} = \dfrac{12}{100}$ → $X = \$1.80$	$\$1.80 + \$14.99 =$ $\$16.79$
b)	New car	$7 890.00		
c)	40" flat screen tv	$699.00		
d)	Tent	$168.79		
e)	Couch	$348.00		
f)	Dehumidifier	$78.99		
g)	Drill and bit set	$248.99		
h)	Shoes	$79.98		

4) Solve these percent problems

    a) All pants are 25% off said the sign in the window. What will be the sale price of a pair of pants originally priced at $59.78?

    b) 30% of the forest will be cut down in the next three years. The forest is 150 acres large. How many acres will be cut?

    c) All winter clothes are on sale for 15% off. A set of gloves cost $18.00. How much will you **save**?

    d) 15% of the staff will be laid off at the mill next month. There are 180 employees. How many will be let go?

    e) Vancouver has grown in population from 2001 to 2006. In 2001 it had a population of 1 986 985, and went up 6.5%. What was the population in 2006?

f) Kelowna's population in 2001 was 147 739, it grew 9.8% in 5 years. What was its
    population in 2006?

g) Rent will increase 10% on Oliver's apartment on January 1. His rent was $460.00 per
    month. What will his new rent be?

h) The French Bread Bakery staff will get a one-time 15% pay increase on their Christmas
    pay check. If Manon works 63 hours at $12.78/hour for that pay check, how much will
    she get paid? (figure out the regular pay first and then add on the 15% increase)

i) A real estate agent sells a home for $239 000.00. How much is the commission at 7%?

j) Sylvia receives a weekly salary of $280.00 plus a commission of 8% on all sales she
    makes over $2000. Last week she sold $5 600.00 worth of merchandise. How much
    money did she make?

k) The bill at the restaurant was $38.00. Add a 20% tip. What did the customer pay in total?

l) Mica's wage increase in 15% after her first year of training. She made $10.90 per hour for her first year. What will her wage be after the increase? (round to the nearest cent)

m) The mark up on a pair of fancy running shoes is 45%. The shoes cost the buyer $38.00. How much will they be sold for after mark-up?

n) A house is insured for 90% of its value against damage. How much money will the owner get back on a destroyed house that is worth $370 000.00?

o) Julie and Frank both work at the local restaurant. Julie is a cook's assistant and Frank is the cook. Julie makes $9.00 an hour, Frank makes $12.25 an hour. The boss is giving them both a raise of 8%. What will their new wages be? (remember to round to the nearest cent)

p) 15 employees work at the local day care. There are 4 pay categories. The board of directors has managed to find enough money to give each employee a 9% raise. Use the chart to figure out the following new pay rates.

Category	Pay rate	9% increase	New pay rate
1	$12.15		
2	$13.50		
3	$14.00		
4	$15.50		

Answers to Review

1)

a) 11	b) 9	c) 10.5	d) 6	e) 3.2	f) 7.2
g) 1.2	h) 2.25	i) 6.5	j) $140.25	k) 0.05	l) 0.61
m) $156.00	n) 2.1				

2)

a) 9	b) 6.5	c) 5.44	d) 67.5	e) 8.4	f) 2.7
g) 0.75	h) 27.1$\overline{6}$				

3)

b) $946.80, $8 836.00   c) $83.88, $782.88   d) $20.25, $189.04

e) $41.76, $389.76   f) $9.48, $88.47   g) $29.88, $278.87

h) $9.60, $89.58

4)

a) $44.84	b) 45 acres	c) $2.70	d) 27 people	e) 2 116 139	f) 162 217
g) $506.00	h) $925.91	i) 16 730	j) $568.00	k) $45.60	l) $12.54
m) $55.10	n) $333 000.00	o) Julie $9.72 Frank $13.23			

p)

Category	Pay rate	9% increase	New pay rate
1	$12.15	$1.09	$13.24
2	$13.50	$1.22	$14.72
3	$14.00	$1.26	$15.26
4	$15.50	$1.40	$16.90

TEST TIME!

Ask your instructor for the Practice Test
for this unit.

Once you've done the practice test,
you need to do the unit 4 test.

Again, ask your instructor for this.

Good luck!

Unit 4

More Working with Percent

Topic A: Finding What Percent one Number is of Another

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

In problems where you must find what percent one number is of another, the **missing term** is the **percent**. You will be told the part (is) and the whole (of), you know the 100, and you solve for the missing percent.

Example A: 4 is what percent of 5?

4 is the part (is)

5 is the whole (of)

% is unknown (call it P)

Write the proportion
$$\frac{4}{5} = \frac{P}{100}$$

Cross multiply to solve

$400 = 5P$

$400 \div 5 = P$

$P = 80\%$ Be sure to write the percent sign %.

In percent problems, the number after "of" usually is the whole.

The number close to "is" usually is the part. You may find it helpful to think "is over of $\left(\dfrac{\text{is}}{\text{of}}\right)$". An = can substitute for "is".

12 is what percent of 15?

$$\frac{is}{of} \text{ will help to find } \frac{part}{whole}$$

$$\frac{12 \ (is)}{15 \ (of)} = \frac{N}{100}$$

Example B: What percent of 85 is 60?

$60 =$ the part (close to "is")

$85 =$ the whole (after "of")

$\% =$ the unknown

proportion $\left(\dfrac{is}{of}\right)$ $\qquad \dfrac{60}{85} = \dfrac{P}{100}$

simplify $\qquad \dfrac{\cancel{60}^{12}}{\cancel{85}_{17}} = \dfrac{P}{100}$

cross multiply and solve

$1\ 200 = 17P$

$1\ 200 \div 17 = P$

$P = 70.588\% \qquad$ round to 70.6%

Exercise One

The following examples ask you to find what percent one number is of another. The **missing term** is the **percent**. Look carefully at the wording and decide which number is the part (close to "is") and which number is the whole thing (after "of").

Write the proportion but <u>do not</u> solve the problem

a) 3 is what % of 6?

b) 12 is _____ % of 5?

c) What % of 27 is 9?

d) What % of $\frac{1}{2}$ is $\frac{1}{4}$?

e) _____ % of 50 is 25?

f) _____ % of 64 = 48

Exercise Two

Solve each question by first setting up the proportion
$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100} \, .$$

a) 25 is _125_ % of 20. $\dfrac{\cancel{25}^{5}}{\cancel{20}_{4}} = \dfrac{P}{100}$

$$500 = 4P \quad 500 \div 4 = P \quad P = 125\%$$

b) 3 is what percent of 60?   _____

c)   3 is what percent of 4?   _____

d)   1 is what percent of 3?   _____

e)   What % of 50 is 35?   _____

f)   What percent of 350 is 42?   _____

g)   15 is ____% of 12.   _____

h)   14 is ____% of 700.   _____

i)   What percent of 96 is 12?   _____

j)   2 is ____% of 125.   _____

k)   7 is what percent of 8?   _____

l)   46 is ____% of 40.   _____

Answers to Exercise Two

b) 5 %	c) 75 %	d) $33.\overline{3}$ %	e) 70 %
f) 12 %	g) 125 %	h) 2 %	i) 12.5 %
j) 1.6 %	k) 87.5 %	l) 115 %	

Exercise Three   Solve the following by setting up the proportion.

a)   16 is _____% of 64.

b)   17 is _____% of 85.

c)   What % of 52 is 13? _____

d)   What percent of 65 is 39? _____

e)   9 is _____% of 54.

f)   33 is _____% of 200.

g) What percent of 125 is 75? _____

h)   1 is _____% of 200.

i)   36 = _____% of 12

j)   _____% of 72 = 27

k)   35 is what % of 42?   _____

l)   _____% of 48 = 18

m)   125 = _____% of 75

n)   What % of 18 = 24? _____

Answers to Exercise Three

a) 25 %   b) 20 %   c) 25 %   d) 60 %

e) $16.\overline{6}$ %   f) 16.5 %   g) 60 %   h) 0.5 %

i) 300 %   j) 37.5 %   k) $83.\overline{3}$ % or $83\frac{1}{3}$ %   l) 37.5 %

m) $166.\overline{6}$ %   n) $133.\overline{3}$ % or $133\frac{1}{3}$ %

Finding the Percent of an Increase or Decrease

You learned in to find the amount of an increase (gain) or decrease (loss) when given the percent of the increase or decrease.

Now you are going to find the percent of the increase or decrease when you are given the amounts. This is called the rate of the increase or decrease.

Problems which ask you to find the percent of increase or decrease often involve two steps:

Step 1   Find the amount of change (either increase or decrease) by finding the difference between the two amounts given. Subtract to find the difference.

Step 2   Find the percentage of increase/decrease. Always compare the change (amount of increase or decrease) to the amount before the change (the original amount) using this proportion.

$$\frac{\text{amount of increase or decrease}}{\text{original amount}} = \frac{P}{100}$$

Example A: The rent went from $375 a month to $427.50 a month. What is the percent of the increase?

Step 1   Find the change (the amount of increase) by finding the difference between the amounts.

$427.50 - 375 = $52.50
The amount of increase is $52.50

Step 2   Find the % of increase.

The amount of increase is $52.50

The original amount (the amount before the increase) is $375.

What % of $375 is 52.50?

$$\frac{X}{100} = \frac{52.50}{375} \longrightarrow X\ 375 = 5250 \longrightarrow$$

$$X = \frac{5250}{375} \longrightarrow X = 14\%$$

The rent increase is 14%.

Example B:

The hours of operation at the college were reduced from 35 hours a week to 30 hours a week. What is the percent of this cut in operations?

Step 1   Find the amount change (a decrease) by finding the difference between the amounts.

35 hours - 30 hours = 5 hours

The amount of decrease is 5 hours.

Step 2   Find the % decrease.

Decrease is 5 hours.

Original amount is 35 hours.

What percent is 5 of 35?

$$\frac{5}{35} = \frac{P}{100} \longrightarrow P\ 35 = 500 \longrightarrow \frac{500}{35} = P$$

$$P = 14\frac{2}{7}\ \%$$

The hours of operation at the college were cut $14\frac{2}{7}$%.

Exercise Four

Solve the following problems.

a) Ms. Lister's bi-weekly unemployment cheque increased from $405 to $435. What percent increase is this?

b) The worker's wage recently went from $10.00 an hour to $10.50 an hour. What is the percent of this increase?

c) Joan weighed 72 kg before she went on a programme of strict exercise and careful eating. She now weighs 60 kg. What is the percent of her weight loss?

d) The car dealership gives a special deal if the customer does not have a trade-in and pays cash. The dealers will only charge $10 650 for a car listed at $12 000. What is the percent savings in this deal?

e) The enrolment in the afternoon bowling league went from 40 bowlers to 50 bowlers for the spring session. What is the percent of this increase in bowlers?

f) A regular toilet uses 20 litres of water per flush. By purchasing a new low flow toilet, the water use is 6 litres per flush. What is the percent savings of water per flush if the new tank is used?

g) Garbage has become a major problem all over the world. In Canada, each person produces an average of 2 kilograms of garbage per day! A landfill that takes the garbage of 20 000 people therefore receives an average of 40 000 kg of garbage per day. Although organic household waste like vegetable peelings can be composted at home making new soil, about 7 600 kg of household organic waste goes to that landfill site every day. If all the people using that landfill composted their household organic waste, by what percent would the total amount of garbage be reduced?

h) Sixteen litres of raw strawberries only produced 13.5 litres of cooked strawberries. What is the percent of the shrinkage of these strawberries?

Answers to Exercise Four

a) 7.4 % increase	b) 5 % increase	c) 16.$\overline{6}$ % decrease
d) 11.25 % savings	e) 25 % increase	f) 70 % savings
g) 19 % reduction	h) 15.625 % shrinkage	

Other Problems

Many situations compare one number to another.

- 24 out of 25 on the test
- 6 out of 10 people are overweight
- the government spends 27¢ of every federal tax dollar on the national debt

These numbers are often more easily thought about if written as a percent.

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

The following problems ask you to find what percent one number is of another. Often several steps are involved to calculate the part (as in question d) or to calculate the whole (as in question e) You may be asked to use the % after you find it (as in questions c and g). Remember the whole thing = 100%.

Exercise Five   Solve the following problems.

a) The Doal family net income is $2300 per month. Their mortgage payment is $750 each month. What percent is the mortgage payment of their monthly income?

b) The town of 20 000 people who use the landfill sight were asked to vote in a referendum concerning a recycling depot. The town had 12 000 eligible voters and 7 500 people voted in the referendum. What percent was the voter turnout? (Voter turnout is the number of people who voted compared to the number of eligible voters. This is usually expressed as a percent.)

c) Jean played on the college volleyball team and missed a lot of classes when she travelled to tournaments. She missed nine of the 42 English classes last semester.

    i)   What percent of her English classes did she miss?

    ii)   What percent of her English classes did she attend?

d) If a bank loan including interest totalled $4 200, and 3 payments of $210 have been made, what percent of the money owed has been paid back? (2 steps)

e) Four women and six men serve on the Village Council. What percent of the council members are women?

f) Gail bought a $500 G.I.C. (Guaranteed Investment Certificate) one year ago. She was delighted to receive her annual interest cheque of $52.50 today. What percent interest did Gail's G.I.C. pay for that year?

g) If a math book had 320 pages and you still had 110 pages left to do, what percent of the book had you finished? (2 steps)

Answers to Exercise Five

a) 32.6 % of monthly income　　　　　　　　b) 62.5 % voter turnout

c) i) 21.4 % of English classes   ii) 78.6 % of English classes

d) 15 % of money owed　　　　　　　　　　e) 40 % of members are women

f) 10.5 % interest　　　　　　　　　　　　g) 65.625 % or $65\frac{5}{8}$ % of the book

School Grades

When looking at test results, the mark shows how you did on the test.

- If you get $\dfrac{7}{10}$ on a test, you know you got 7 answers right, and 3 answers wrong.

Sometimes it is also helpful to see your mark as a percentage.

Example A:

$\dfrac{7}{10} = \dfrac{N}{100}$ $\longrightarrow$ By solving for N, the percentage can be found.

$7 \times 100 = N \times 10$ $\longrightarrow$ $\dfrac{70\cancel{0}}{\cancel{10}} = \dfrac{N\cancel{10}}{\cancel{10}}$ $\longrightarrow$ $70 = N$

So, $\dfrac{7}{10} = 70\%$

Now, you can see that the test mark of $\dfrac{7}{10}$ equals 70%.

Example B:

The test result was $\dfrac{15}{43}$, what was the percent on the test?

$\dfrac{15}{43} = \dfrac{N}{100}$ $\longrightarrow$ $15 \times 100 = N \times 43$ $\longrightarrow$ $\dfrac{1500}{43} = \dfrac{N\cancel{43}}{\cancel{43}}$

$N = 34.88\ \%$

If you round $N = 35\%$

Not such a great mark!

Example C:

Find the percent of the following grade: $\dfrac{89}{97}$

$\dfrac{89}{97} = \dfrac{P}{100}$ $\longrightarrow$ $89 \times 100 = P \times 97$ $\longrightarrow$ $\dfrac{8900}{97} = \dfrac{P\cancel{97}}{\cancel{97}}$

$P = 91.75\ \%$ or 92%

Exercise Six

Find the percents for the following test grades. Round your answer to the nearest percent.

a) $\dfrac{33}{42}$ _____

b) $\dfrac{24}{40}$ _____

c) $\dfrac{13}{21}$ _____

d) $\dfrac{5}{10}$ _____

e) $\dfrac{90}{120}$ _____

f) $\dfrac{100}{110}$ _____

g) $\dfrac{7}{11}$ _____

h) $\dfrac{10}{20}$ _____

Answers to Exercise Six

a) 79%	b) 60%	c) 62%	d) 50%	e) 75%
f) 91%	g) 64%	h) 50%		

Topic A: Self-Test

Mark   /23   **Aim 19/23**

A. Solve to find the missing percents.   7 marks

a)   12 is _____% of 60.

b)   _____% of 32 = 8.

c)   _____% of 50 = 70.

d)   15 is _____% of 75.

e) $3\frac{1}{2}$ is _____% of 70.

f)   8.2 = _____% of 32.8

g)   What percent of 64 is 48?   _____

B. Problems. 6 marks

a) The $140 jacket was on sale for $126. What percent is the savings?

b) The rent on the apartment went from $320 a month to $400 dollars a month. What percent is this rent increase?

c) The grocery bills were $87.50, $22.50 and $30.25 last week. The net weekly income for that family is $530. What percent of the income was spent on groceries?

C. School Grades 10 marks

Find the percents for the following grades on tests. Round your answer to the nearest whole percent.

a) $\dfrac{3}{15}$ b) $\dfrac{7}{15}$

c) $\dfrac{12}{15}$ d) $\dfrac{10}{19}$

e) $\dfrac{71}{92}$

f) $\dfrac{132}{140}$

g) $\dfrac{13}{20}$

h) $\dfrac{19}{20}$

i) $\dfrac{5}{7}$

j) $\dfrac{35}{40}$

Answers to Topic A Self-Test

Part A

a) 20 %	b) 25 %	c) 140 %	d) 20 %
e) 5 %	f) 25 %	g) 75 %	

Part B

a) 10 % savings	b) 25 % increase	c) 26.5 % of weekly income

Part C

a) 20%	b) 47%	c) 80%	d) 53%
e) 77%	f) 94%	g) 65%	h) 95%
i) 71%	j) 88%		

Topic B: Finding a Number when a Percent of it is Given

$$\frac{\text{is (part)}}{\text{of whole}} = \frac{\%}{100}$$

In problems when a certain percentage of a number is given, the **missing term is the whole**. You will be told the % and the part (is), and asked to find the whole (of), which is 100%.

Example A: 20% of what number is 14?

The part = 14.

The whole ("what number") = unknown (call it N).

The percent = 20%.

Write the proportion
$$\frac{14}{N} = \frac{20}{100}$$

Solve the proportion

simplify $\dfrac{14}{N} = \dfrac{\cancel{20}^{1}}{\cancel{100}_{5}}$

cross multiply

$14 \times 5 = N \times 1$

$70 = N$

20% of _70_ = 14

Check by finding 20% of 70. The answer should be 14.

$\dfrac{20}{100} = \dfrac{N}{70}$ $\longrightarrow$ $N100 = 140$ $\longrightarrow$ $N = 14$

Example B: $33\frac{1}{3}\%$ of ____ is 60.

    part = 60

    whole = N

    percent = $33\frac{1}{3}\% = \frac{1}{3}$

    proportion

$$\frac{60}{N} = \frac{1}{3} \longrightarrow 3 \times 60 = 1N \longrightarrow N = 180$$

    $33\frac{1}{3}\%$ of _180_ is 60.

To check the answer, find $33\frac{1}{3}\%$ of 180. The answer should be 60.

Example C: $12 = 15\%$ of what number?

$$\frac{12}{N} = \frac{15}{100} \longrightarrow \frac{12}{N} = \frac{\cancel{15}^{3}}{\cancel{100}_{20}} \longrightarrow 12 \times 20 = 3N$$

$$240 = 3N \longrightarrow 240 \div 3 = N \longrightarrow 80 = N$$

$$12 = 15\% \text{ of } \underline{80}$$

To check, find 15% of 80. The answer should be 12.

Example D: 24 is 40% of what number?

$$\frac{24}{N} = \frac{40}{100} \longrightarrow \frac{24}{N} = \frac{\cancel{40}^{2}}{\cancel{100}_{5}} \longrightarrow 24 \times 5 = N \times 2$$

$$120 = 2N \longrightarrow 120 \div 2 = N \longrightarrow N = 60$$

24 is 40% of 60.

Exercise One

Set up the proportion. <u>Do Not</u> solve the question.

a) 18 is 50% of what number? $\underline{\qquad\qquad}$

b) 24 is 15% of what number? $\underline{\qquad\qquad}$

c) 90 is 4% of what number? $\underline{\qquad\qquad}$

d) $15 = 60\%$ of what number? $\underline{\qquad\qquad}$

e) $12\frac{1}{2}\%$ of what number is 2? $\underline{\qquad\qquad}$

f) 200% of what number is 86? $\underline{\qquad\qquad}$

g) 75% of what number $= 10$? $\underline{\qquad\qquad}$

h) $66\frac{2}{3}\%$ of what number $= 500$? $\underline{\qquad\qquad}$

i) 37.5% of $\underline{\quad} = 240$. $\underline{\qquad\qquad}$

j) 10% of $\underline{\quad}$ is \$25. $\underline{\qquad\qquad}$

Answers to Exercise One

a) $\dfrac{18}{P} = \dfrac{50}{100}$ b) $\dfrac{24}{N} = \dfrac{15}{100}$ c) $\dfrac{90}{X} = \dfrac{4}{100}$ d) $\dfrac{15}{N} = \dfrac{60}{100}$ e) $\dfrac{2}{F} = \dfrac{12\frac{1}{2}}{100}$

f) $\dfrac{86}{P} = \dfrac{200}{100}$ g) $\dfrac{10}{Y} = \dfrac{75}{100}$ h) $\dfrac{500}{P} = \dfrac{66\frac{2}{3}}{100}$ or $\dfrac{2}{3}$ i) $\dfrac{240}{L} = \dfrac{37.5}{100}$ j) $\dfrac{10}{N} = \dfrac{10}{100}$

Exercise Two

Solve the following. Check your answers to see if your proportion was set up correctly.

a) 60 is 75% of what number?   _____

b) 950 is 95% of   _____

c) $125 = 33\frac{1}{3}$ % of what number?   _____

d) $87\frac{1}{2}$ % of what number is 280?   _____

e) 50% of _____ = 95.   _____

f) 25% of _____ is 800.   _____

g) 12.5% of what number is 64?   _____

h) 120% of _____ is 6.   _____

i) 270 is 100% of what number?   _____

j) $3\frac{1}{2}$ % of what number is 21?   _____

Answers to Exercise Two

a) $\dfrac{60}{X} = \dfrac{75}{100}$, 80   b) $\dfrac{950}{P} = \dfrac{95}{100}$, 1 000   c) $\dfrac{125}{N} = \dfrac{33\frac{1}{3}}{100}$, 375   d) $\dfrac{280}{L} = \dfrac{87\frac{1}{2}}{100}$, 320

e) $\dfrac{50}{100} = \dfrac{95}{X}$, 190   f) $\dfrac{25}{100} = \dfrac{800}{P}$, 3 200   g) $\dfrac{12.5}{100} = \dfrac{64}{N}$, 512   h) $\dfrac{120}{100} = \dfrac{6}{J}$, 5

i) $\dfrac{100}{100} = \dfrac{270}{N}$, 270   j) $\dfrac{3\frac{1}{2}}{100} = \dfrac{21}{P}$, 600

Exercise Three   Solve the questions.

a) 480 is $66\frac{2}{3}$ % of _____.

b) 40% of _____ is 50.

c) 25% of what number is 8? _____

d) 12 is 75% of what number?   _____

e) $33\frac{1}{3}$ % of what number is 99?   _____

f) 2800 is $87\frac{1}{2}$ % of what number.   _____

g) 97% of what number is 970?   _____

h) 3 is 150% of what number?   _____

i) 122 is 80% of _____.

j) $8.75 is $10\frac{1}{2}$ % of what number?   _____

Answers to Exercise Three

a) 720	b) 125	c) 32	d) 16
e) 297	f) 3 200	g) 1 000	h) 2
i) 152.5	j) $83.33		

Solving Problems when the Percent of a Number is Given

Read the problems carefully. More than one step may be needed. Look at the wording so you will recognize problems missing the whole and be able to tell them from problems missing the part.

Exercise Four

Solve the problems. Round money to the nearest cent.

a) When the business was declared bankrupt, all the creditors (people owed money by the business) were paid 55% of the money owed them.

    i)   If a creditor received $8 000 from the bankrupt business, how much money had he really been owed?

    ii)   How much money did the creditor lose on this business?

b) In telethons and other fund-raising events, records show that about 80% of the money pledged is actually collected. The local telethon organizers need to raise $12 000. To actually raise $12 000, their goal for pledges should be what amount?

c) Statistics suggest that only about 46% of victims of sexual assault report the assault to the police. If a police department received 920 reports of sexual assault in a year, how many cases of sexual assault may really have occurred?

d) The shoe store sold all merchandise at 25% off in a huge clearance sale. They took in $3 500 in the first day of the sale. If the same shoes had been sold at the regular price, how much money would they have taken in? (Note—this problem has 2 steps. The merchandise was 25% off, so it sold for 100% - 25% = 75% of the original price.)

e) Marie paid $4 500 for a second hand car in the Bonanza Summer Sale when all cars were $33\frac{1}{3}$ % cheaper than their list price. What had been the listed price of Marie's car? (Watch for the extra step!)

f) A 15% down payment is required on clothes before any alterations will be done for you by the store tailor. If someone had to make a down payment of $30 before his suit would be altered, what was the price of the suit?

g) The quorum (number of people who must be present so official business can be decided) set in a particular club's bylaws is 30% of the members. They had exactly a quorum at their last meeting when 12 people came. How many members are in this club?

h) A common rate of commission earned by real estate agents is $3\frac{1}{2}$ %. If an agent had a gross income of $63 000 from commissions in one year, what was the value of the houses sold?

i) The distance from Vancouver to Osoyoos is 420 km. This is 60% of the total distance of the trip from Vancouver to Nelson. What is the distance from Vancouver to Nelson?

Answers to Exercise Four

a) i) $14 545.45   a) ii) $6 545.45   b) $15 000.00   c) 2 000 cases
d) $4 666.67   e) $6 750   f) $200   g) 40 members
h) $1 800 000   i) 700 km

Topic B: Self-Test

A. Solve these questions: **6 marks**

a)   2.5% of what number is 160?   _____

b) $37\frac{1}{2}$% of what number is 24?   _____

c)   5 is $\frac{4}{5}$% of _____.

d)   20 is $12\frac{1}{2}$% of what number?   _____

e)   180 is 90% of what number?   _____

f)   28 is 35% of what number?   _____

B. Problems.   **4 marks**

a) If a bank insists that new house buyers have a cash down payment of 12%, what house price can a couple afford if they have saved a $15 000 down payment?

b) Jim has really cut down on his smoking. He now smokes 7 cigarettes a day, which he says is only 20% of what he used to smoke. How many cigarettes a day did Jim smoke before he started cutting down?

Topic B: Self-Test

Part A

a) 6 400	b) 64	c) 625	d) 160
e) 200	f) 80		

Part B

a) $125 000   b) 35 cigarettes per day

Unit 4 Review

You have been practising three types of percent problems. You have learned that one proportion can be used to solve all the problems:

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

A: Finding a Percent of a Number

- You are given the percent and the whole.

- The missing term is the part (call it **N**).
$$\frac{N}{\text{whole}} = \frac{\%}{100}$$

B: Finding What Percent One Number is of Another

- You are told the part and the whole.

- The missing term is the percent (call it **P**).
$$\frac{\text{part}}{\text{whole}} = \frac{P}{100}$$

C: Finding a Number When a Percent of It Is Given

- You are given the part and the percent.

- The missing term is the whole (call it *W*).
$$\frac{\text{part}}{W} = \frac{\%}{100}$$

Real-life situations and real math problems often require several steps to collect and organize all the information. Look for those extra steps in the problems that follow. When you read the problems look for the part, the whole and the percent. Decide which term is missing. Once you know which term is missing the problem can be solved by using the proportion $\dfrac{\text{part}}{\text{whole}} = \dfrac{\%}{100}$ or the appropriate short method.

Solve these problems using proportion. **Write all your work with the problem so your instructor can help you should you have any difficulty**. Remember to check that the answer makes sense and to write a sentence answer. For these problems, round your answers this way:

- percents to the nearest tenth of a percent

- money to the nearest cent

- decimal fractions to the nearest thousandth

a) Derek is driving home from college, a distance of 240 km. He has stopped for a quick lunch after driving 180 km. What percent of his trip home does he have left to drive?

b) You need a minimum of 80% on the test which means you must get at least 36 marks. What are the total possible marks on the test?

c) Canadian residents pay a 5% surtax on their federal income tax. The 5% surtax is an extra tax on federal income tax. If a person has to pay $2250 federal income tax in a year, how much will the surtax be?

d) The Lings are buying a $130 000 home and have made a $19 500 down payment. What percent of the purchase price is their down payment?

e) Ann and Joe bought their home twelve years ago and have paid 45% of the principle amount of their mortgage. They have paid $18 000 towards the principle of the mortgage. What was the principle amount of the mortgage to start with?

f) The waiters at the restaurant must contribute money to be shared among the cocktail servers and kitchen staff. Each waiter contributes 4% of his or her total food and drink sales. Craig's total sales were $645 in his 5 hour shift.

   i)   How much money must he contribute to the kitchen and cocktail servers?

   ii)   Craig made 12% in tips on his sales tonight. What amount were his tips?

   iii) Craig is paid $9.50 an hour. Calculate how much money he will receive for his evening's work. Include wages and tips. Remember to subtract the money he had to pay to the kitchen and cocktail staff.

g) The college employs 432 people. Administrators make up 12% of the staff. All other employees belong to unions. How many employees belong to unions?

h) The cost of hydroelectric power for our home last year was 210% of what it was six years ago. Last year our power bill totalled $960. How much was it six years ago?

i) The graduating class had 68 women and 80 men. What percent of the class were men?

j) $16\frac{2}{3}$ % of the tickets for the rock concert were sold in the first hour the telephone order lines were open. In that hour, 2 500 tickets were sold. What was the total number of tickets available for the concert?

k) Mrs. Brown went shopping for her daughter. At one store, she bought her daughter a purse for $39.99, a pair of shorts on sale for $18.50, a great looking shirt for $29.95 and some shampoo for $4.25.

    i)   Calculate the Harmonized Sales Tax (12%) on these purchases.

    ii)   Calculate the total cost, including taxes, for all these items.

l) The total student enrolment in the school district has increased from 18 506 students to 19 724 students in the last year. What is the percent of this increase in student enrolment?

m) Al bought his second hand off-road motorcycle for $1500 and sold it three years later for $1175. By what percent did his motorcycle depreciate (decrease in value)?

n) Pat operates a street-vendor's cart selling hot dogs, sausages on a bun and soft drinks. The basic pay is $100 per week and 28% commission on all sales over $450 in a week. Pat sold $1244 of food and soft drinks last week. Calculate Pat's earnings from the street-vending cart for the week.

o) The 1500 blouses purchased by the large retail chain of ladies' clothing stores cost the company a total of $24 000. The blouses were then priced to sell at $45 each. What is the percent of the mark-up on these blouses? (Hint-First calculate the company's cost price for each blouse.)

p) 92% of the members of the union voted to strike; that is, 320 union members voted to strike. How many members are in the union?

q) Maureen was happy to see that she got 27 out of 30 on her English essay. What percent did she get?

r) The ski jackets were on the summer clearance rack marked "45% off".

    i) What is the sale price of a jacket priced regularly at $229.95?

    ii) What is the total cost of this jacket with H.S.T (12%)?

s) Tuition fees at the university have increased from $49 per credit hour to $62 per credit hour in the last three years. What is the percent of the tuition increase?

t) Jill bought 3 bottles of liquor in the US. The bill, including US sales tax, was $28.50. Assume the American dollar was $1.08 Canadian. Calculate

  i) The value of the liquor in Canadian funds.

  ii) Duty at 110%

  iii) HST. at 12% (on Canadian value + duty)

  iv) Total cost in Canadian funds.

u) The consignment store will sell your good used women's clothing for you. The store owners take a percentage of the selling price as their fee for service. The consignment charges (the % the store owners keep) are as follows:

coats	45%
dresses and skirts	$33\frac{1}{3}$%
bridal and evening gowns	50%
blouses, jeans, and slacks	25%

Lisa and her daughters did a huge closet clean-out and had the following items sold at the consignment store. For each category, calculate the amount for the store fee and the amount Lisa and the girls received.

	Items	Selling Price	Store Fee	Amount for Lisa & her daughters
i)	Wedding dress	$275		
ii)	2 lovely "prom" dresses at $90 each	2 @ $90 = $180		
iii)	3 dresses at $40 each	3 @ $40 = $120		
iv)	Lisa's winter coat at $120	$120		
v)	4 pairs of outgrown jeans at $10 each	4 @ $10 = $40		
vi)	1 silk blouse for $15	$15		
vii)	6 cotton/polyester blouses for $9 each	6 @ $9 = $54		

v) You have just completed four units of the five units in this book. What percent of the book have you completed?!?!?

Answers to Review

a) 25 % of trip left b) 45 marks c) $112.50

d) 15 % of purchase price e) $40 000 f) i) $25.80

f) ii) $77.40 f) iii) $99.10 g) 380 employees

h) $457.14 i) 54.1 % men j) 15 000 tickets

k) i) $11.12 k) iii) $103.81

l) 6.6 % increase m) 21.7 % depreciation n) $322.32

o) 181.3 % mark-up p) 348 members q) 90 % correct

r) i) $126.47 r) ii) $141.65 s) 26.5 % increase

t) i) $30.78 t) ii) $33.86 t) iii) $7.76

t) iv) $72.40

u)

	Items	Selling Price	Store Fee	Amount for Lisa & her daughters
i)	Wedding dress	$275.	$137.50	$137.50
ii)	2 lovely "prom" dresses at $90 each	2 @ $90 = $180	$90.00	$90.00
iii)	3 dresses at $40 each	3 @ $40 = $120	$40.00	$80.00
iv)	Lisa's winter coat at $120	$120	$54.00	$66.00
v)	4 pairs of outgrown jeans at $10 each	4 @ $10 = $40	$10.00	$30.00
vi)	1 silk blouse for $15	$15	$3.75	$11.25
vii)	6 cotton/polyester blouses for $9 each	6 @ $9 = $54	$13.50	$40.50

v) 80 % Well done!

TEST TIME!

Ask your instructor for the Practice Test
for this unit.

Once you've done the practice test,
you need to do the unit 4 test.

Again, ask your instructor for this.

Good luck!

Unit 5:
Statistics

Introduction and Vocabulary

The word *graph* comes from a Greek word meaning *to write or draw.*

Graphs are a special type of drawing or picture showing how numbers relate to each other. Graphs are a convenient way to organize numbers. You may know the old saying, "One picture is worth a thousand words." Graphs give us a general picture of the information to look at first. The details of the information can then be read from the graph.
In this unit you will practice reading five types of graphs: line graphs, bar graphs, histograms, picture graphs, and circle graphs. You will also learn about reading charts and tables.

Study this vocabulary:

Axis An axis is any straight line used for measuring or as a reference. In graphs, we talk about two axes (axes is the plural of axis).

The **horizontal axis** goes across the page. (You will learn in later math courses that the horizontal axis is the **x axis.)**

The **vertical axis** goes up and down the page. (The vertical axis is the **y axis.)**

A trick to remember the difference between horizontal and vertical:

- The horizon is that line of earth you see as you look out at a view:

- The word **horizontal** comes from the root word **horizon**. The horizon goes from left to right, so does the horizontal line in a graph.

Titles Titles of a graph give important information about the graph and often tell more about what the scale means.

Scale Each axis has a scale. The scale is a series of numbers; beside or below the numbers will be a few words telling what the numbers represent. The numbers we want to place on the graph can be read from the scales. The horizontal scale describes the positions on the horizontal (x) axis. The vertical scale describes the positions on the vertical (y) axis.

Source Graphs often give information that has been collected from other reports or publications. The *source* of the information should be written under the graph. Knowing where the information has come from helps you decide the accuracy of the information in the graph.

Trend The trend is the general direction of events, the general idea of changes that are occurring. For example, there is an upward trend in the cost of houses.

Example:
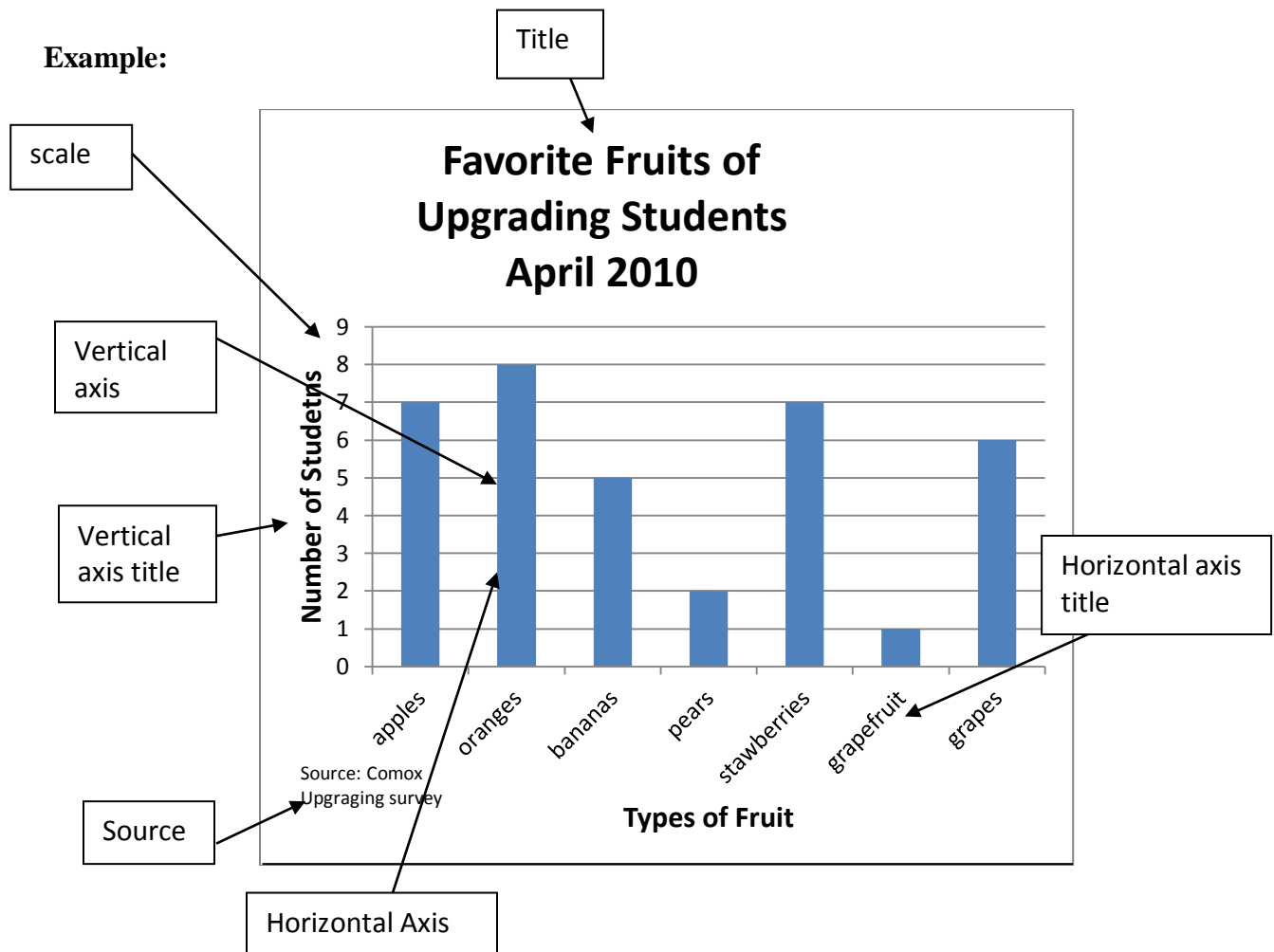
Note: Graphs should always have 0 as the first number on both axes. In this book, there are several graphs that begin at the wrong point. The ability of the Microsoft Excel program used to create these graphs does not allow for creating a special "break" that should be used in situations like in graph 1, 2, & 3 where there would be a lot of wasted space in the graph. If you would like to know more about this issue, please talk to your instructor.

Topic A: Line Graphs

Line graphs are used to show **changes that happen over a period of time.** Line graphs easily show trends and patterns.

The most common way to set up a line graph is to put **time** on the horizontal (x) axis. Whatever is being measured is then put on the vertical (y) axis.

Graph One

Leah's Weekly Basal Heart Rate
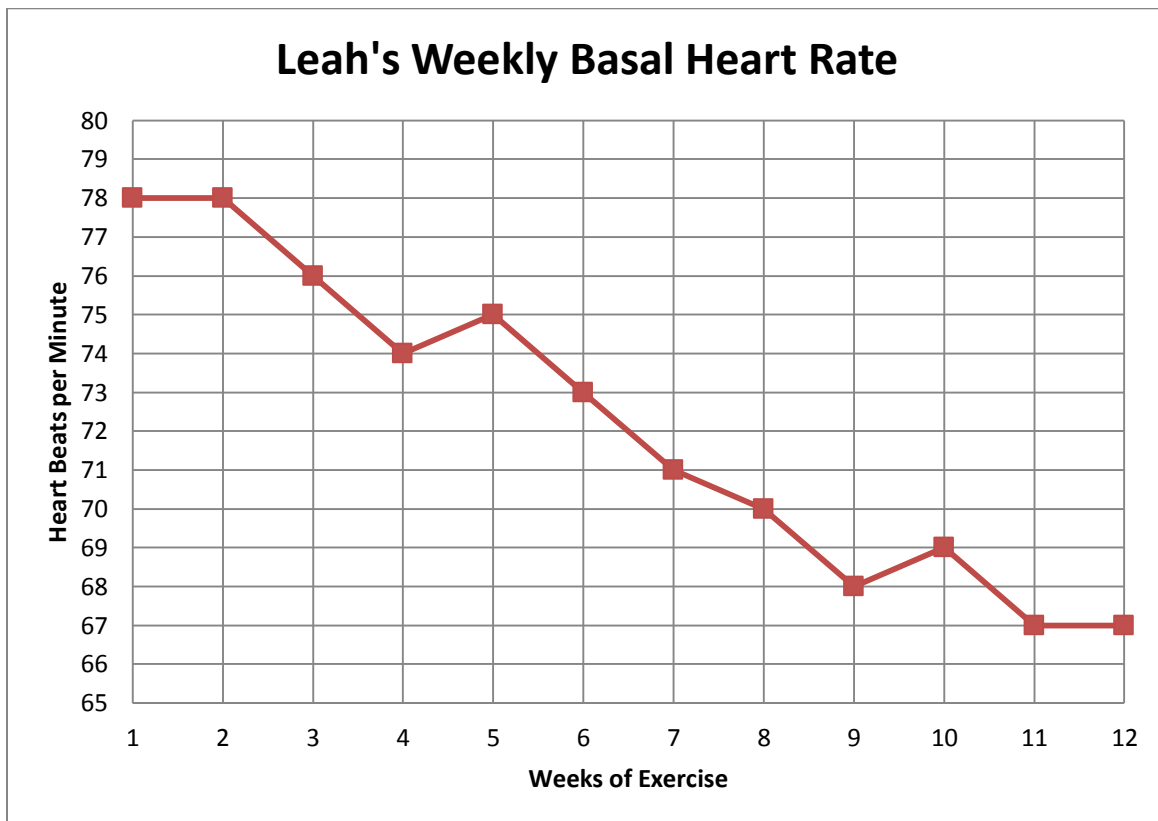
Leah has taken up cycling and jogging to improve her cardiovascular fitness and for weight control. She takes her pulse every Monday morning before she gets out of bed; that is her basal heart rate. Graph One records Leah's heart rate for the first 12 weeks of her exercise program.

a) What is the title of the graph?

_____

b) What is being measured on the vertical axis?

_____

On this particular graph the **vertical scale** is one heartbeat per line. The scale on the **horizontal axis** is one week per line.

c) What was Leah's basal heart rate on the fourth week of her exercise program?

- Find Week 4 on the horizontal (x) axis.

- Look straight up from Week 4 until you come to the point in the graphed line.

- Now look at the scale on the vertical (y) axis. Lay a ruler or straight piece of paper across the graph to help you read the scale at the point for Week 4.

Leah's basal heart rate was 74 beats/min in week four.

d) Find her basal heart rate in Week 7.

_____

e) What does the graph show us happened between Week 9 and Week 10?

_____

f) What **trend** does this graph show?

The graph shows Leah's basal heart rate is "going down" or decreasing.

Often, you need to estimate the value of the point in the graphed line. Look at Graph Two which has Leah's same heart rates recorded.
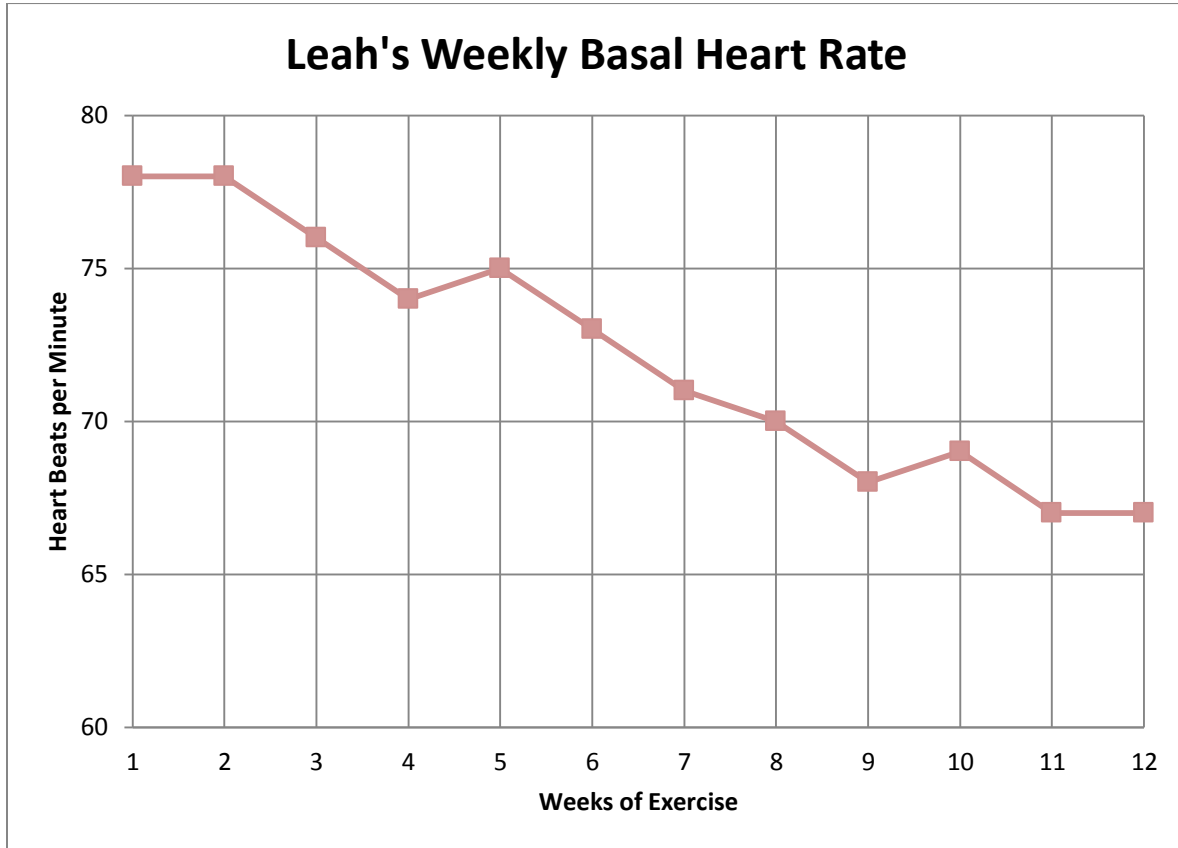
Answers to Graph One
a) Leah's Weekly Basal Heart Rate   b) Heart beats per minute   c) given: 74 beats/min
d) 71 beats/min   e) Leah's heart rate went up from 68 to 69 beats/min

Graph Two

Leah's Weekly Basal Heart Rate

Now the scale on the vertical axis is **five** heartbeats per line. Use a straightedge (ruler or paper) across the graph to help you read the vertical scale.

a) Give Leah's heart rate in Week 5. _____

b) Give Leah's heart rate in Week 10. _____

c) What was Leah's heart rate in Week 12? _____

d) Using the information on the graphs, tell how much Leah's basal heart rate decreased (in beats per minute) from Week 1 to Week 12. _____
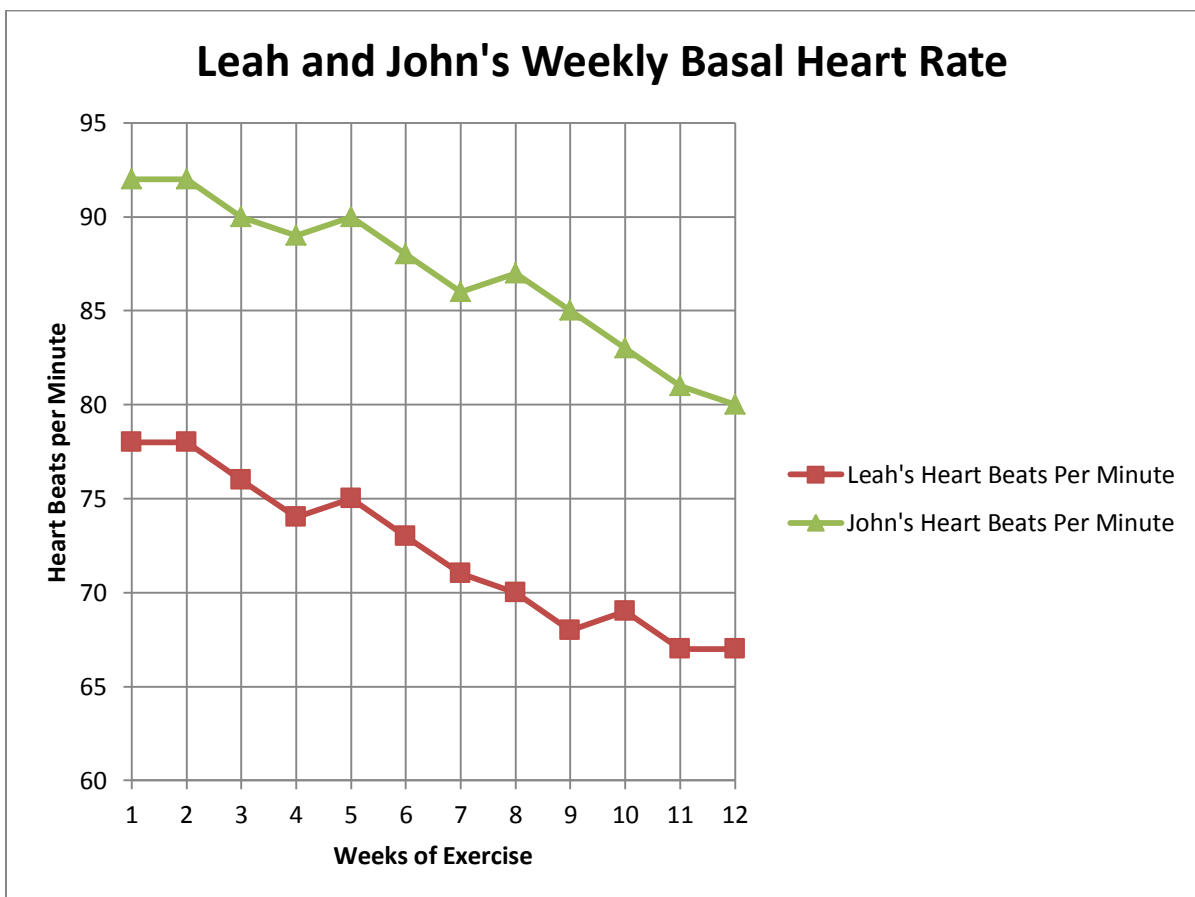
Answers to Graph Two
a) 75 beats/min   b) 69 beats/min   c) 67 beats/min
d) decreased 11 beats/min (78 − 67 = 11)

The person drawing a graph decides how to label it and how to write the scale on the axes depending on the information to be shown on the graph. The graphs about Leah focus on the range of **her** heart rate. There is no need to make the heart rate scale lower than 55 or higher than 80 for Leah.

Graphs often show information about several things on the same graph. Such graphs are very useful for making comparisons. Look for a **legend** or **key** that explains what each graphed line represents. The legend may be printed right by the graphed information or it may be beside or below the graph.

Graph Three

Leah's husband John decided he would exercise as well. He had a rapid heart rate at the beginning of the exercise program. Since his heart rate is higher than 85, we must increase the numbers on the vertical scale so we can graph John's heart rate on the same graph as Leah's.

Leah and John's Weekly Basal Heart Rate

■ Leah's Heart Beats Per Minute
▲ John's Heart Beats Per Minute

a) What was John's heart rate in Week 6? _____

b) What was John's heart rate in Week 10? _____

c) How much lower was Leah's heart rate than John's in Week 8?

_____

d) How much did John's heart rate drop in the 12-week exercise program?

_____

e) What was the amount of change in John's heart rate between Week 3 and Week 7? Was it an increase (+) or decrease (-)?

_____

f) Compare the two graphed lines.

   i) How is the slant of the lines different? _____

       _____

   ii) How are the lines the same? _____

       _____

   iii) You can tell from looking at this graph that John and Leah had a similar trend in the change in their basal heart rates. Did their heart rates increase or decrease? _____

If the graphed lines have about the same slant, the **rate of change** is the same for the information being graphed. By looking at graphed lines you can tell if one has increased or decreased more quickly. You can easily compare changes and tell when the changes occurred.

Steps to Follow When Reading Line Graphs With Two Types of Information

1. Read all the titles so you know what the graph is about.

2. Look at the information on the vertical and horizontal scales.

3. Decide what the graphed lines represent. Look for a legend and be sure you know which line is which.

4. Interpret (read) the information on the graph.
    - First get a general look at what is on the graph.
    - Second, look for the detailed information that you need.

Graph Four

Cross Family's Electricity Costs
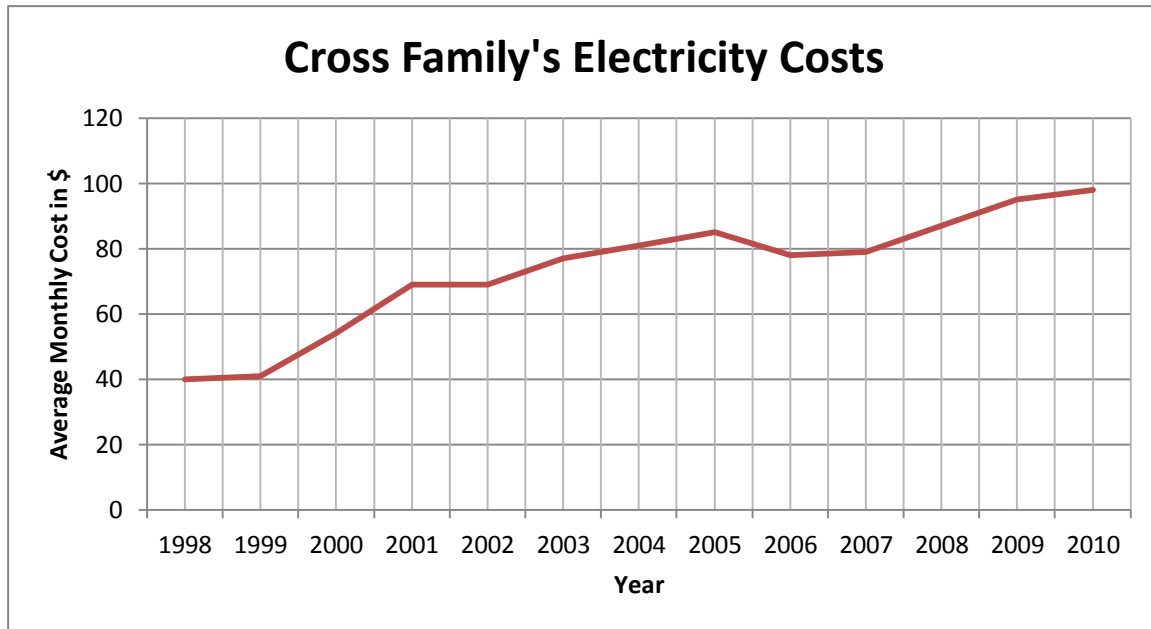
a) What is the general trend in the Cross's average monthly cost for electricity?

_____

b) What was the average monthly cost of electricity in 1998? _____

c) How much more did they pay per month in 2003 than in 2000?

_____

d) What happened to the average cost between 2005 and 2006?

_____

e) By how much did the Cross's average monthly electricity cost increase between 1998 and 2010? _____

Answers to Graph Four

a) increased   b) $40   c) Approximately $23 ($77 – 54)

d) decreased   e) $59

Line graphs can also be used to graph two different types of related information on the same chart. For example, we may have wanted to put Leah and John's weight changes and heart rate changes on the same graph.

When we graph different types of information,

- A sub-title usually explains the two types of information (written in smaller print under the title).

- The horizontal axis is the same.

- The vertical axis on the **right** side of the graph has the scale for the second set of information. For example, on the graph for Leah and John the right vertical axis would be marked in kilograms for weight changes.

Steps to Follow When Reading Line Graphs
With Two Types of Information

1. Read the titles and subtitles.

2. Look at the legend to identify the graphed lines and check where the scale for each line is written (on the left or on the right vertical axis).

3. Go up from the horizontal axis in the same way, and read the appropriate scale on the left or right vertical axis for each graphed line.

Graph Five

Cross Family's Electricity Costs
Average Cost/Month and Annual Kilowatt Hours (kWh) Used
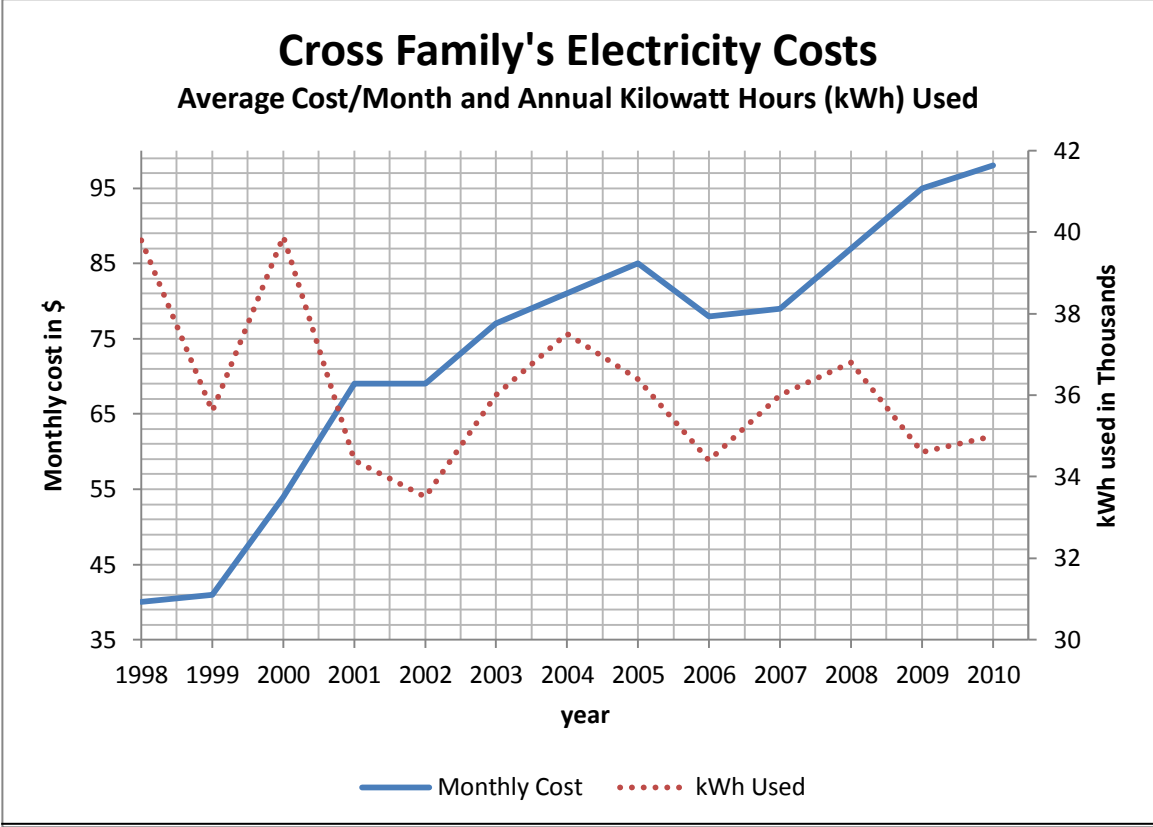
a) What is the title of this graph? _____

b) What is the subtitle? _____

c) Look at the legend. What does the graphed line for "Kilowatt Hours Used" look like?

_____

d) Where is the scale for kilowatt hours? _____

e) How is the scale for kilowatt hours labeled? _____

> *in thousands* means that each figure in the scale is to be multiplied by 1 000.
>
> 32 means 32 000

f) Which two years was the use of kWh the greatest? _____

g) When was the use of kWh the least? _____

h) Look at 1999 and 2000.

   i) Find the amount of kWh used in 1999 _____ and the cost of electricity
   in 1999 _____ .

   ii) Compare to the amount of kWh used in 2000 _____ and the cost of
   electricity in 2000 _____.

   iii) What can you conclude by comparing the cost and the kWh use in 1999 and
   2000?

   _____

i) Now look at 2001 and 2002.

   i) Compare the use of kWh's.   2001 _____ 2002 _____ .

   ii) Compare the cost.   2001 _____ 2002 _____ .

   iii) What can you conclude? _____

j) Between 2009 and 2010 the use of kWh went *up or down*? by _____ kWh.

   The cost of electricity went *up or down*? by $_____ .

Answers to Graph Five

a) Cross Family's Electricity Costs

b) Average Cost/month & Annual Kilowatt Hours Used

c) dotted line   d)   right side of graph   e)   kWh used in thousands

f) 1998 + 2000   g)   2002

h) i) 65 000 kWh and $41.

   ii) 87 000 kWh and $54.

  iii) the price per kWh increased

i)   i) & ii) 2001 ≈ 59 000 kWh & $69; 2002 ≈ 54 000 kWh & $69

iii)   You can conclude the price/kWh decreased.

j)   The use of kWh went <u>down</u> by 2 000 kWh. Cost went <u>up</u> by $5.


⇨   The steepness of the slant of a graphed line gives you a picture of the **rate of change**. The steeper the slant the greater the change.

This double graph shows that the average monthly cost of electricity has increased while the annual use of kilowatt hours has shown an overall decrease.

Topic A: Self -Test

Label all the parts of the following line graph: **7 Marks**

1. Horizontal Axis
2. Vertical Axis
3. Title
4. Scale(s)
5. Source
6. Vertical axis title
7. Horizontal axis title

Number of Births in British Columbia

Source: Statistics Canada, April 2010
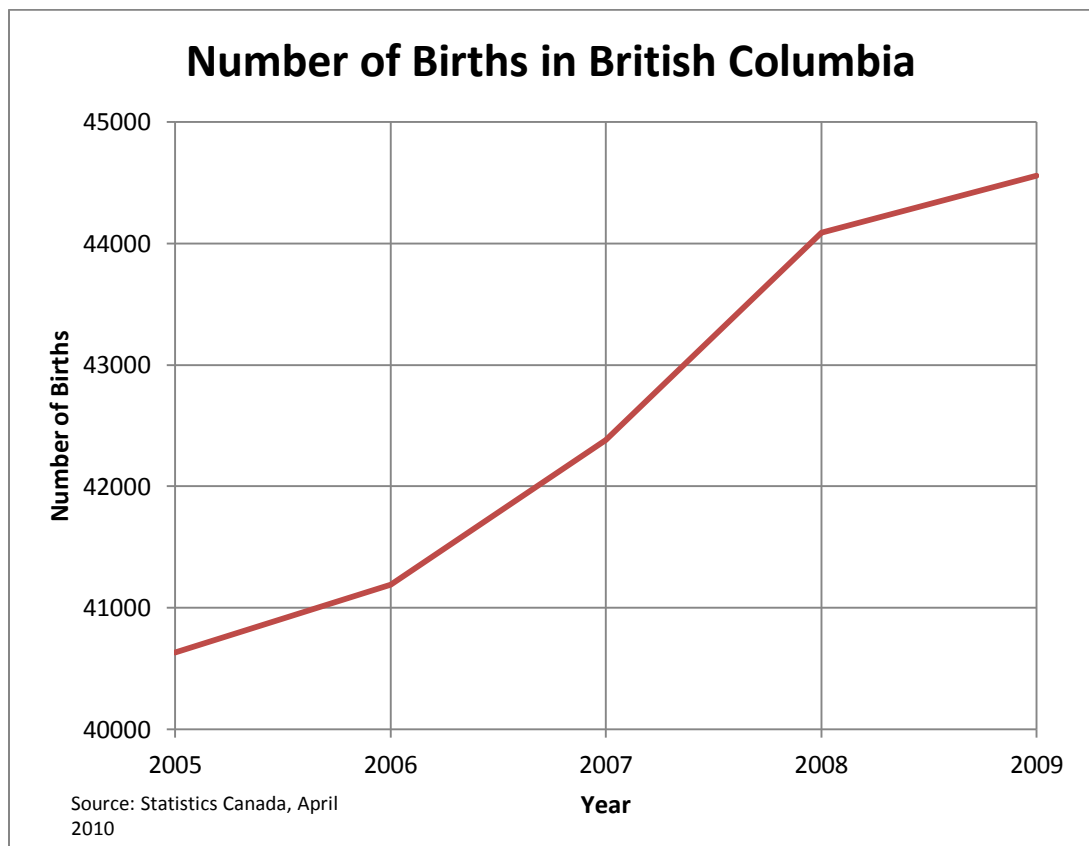
Answer to Self-Test: Go back to the introduction of this unit to check your labeling compared to the example there.


Topic B: Bar Graphs

Bar graphs **compare quantities**. Bar graphs are commonly used to illustrate information in newspapers, in magazine articles, and so on. Bar graphs may be written with the bars arranged vertically or horizontally. Graph One is shown both ways – first with vertical bars and second with horizontal bars.

Steps To Follow When Reading a Bar Graph:

1. Read the title and subtitles so you know what you are looking at.

2. Read the information on the vertical and horizontal axes. Notice that each bar represents a different item.

3. Look carefully at the scale. What unit of measure is being used? The unit of measure will be the same for each bar so that you can compare them.

4. Compare the length or height of each bar to find the information that you want.

Graph One

Lengths of Some British Columbia Rivers

Lengths of Some British Columbia Rivers
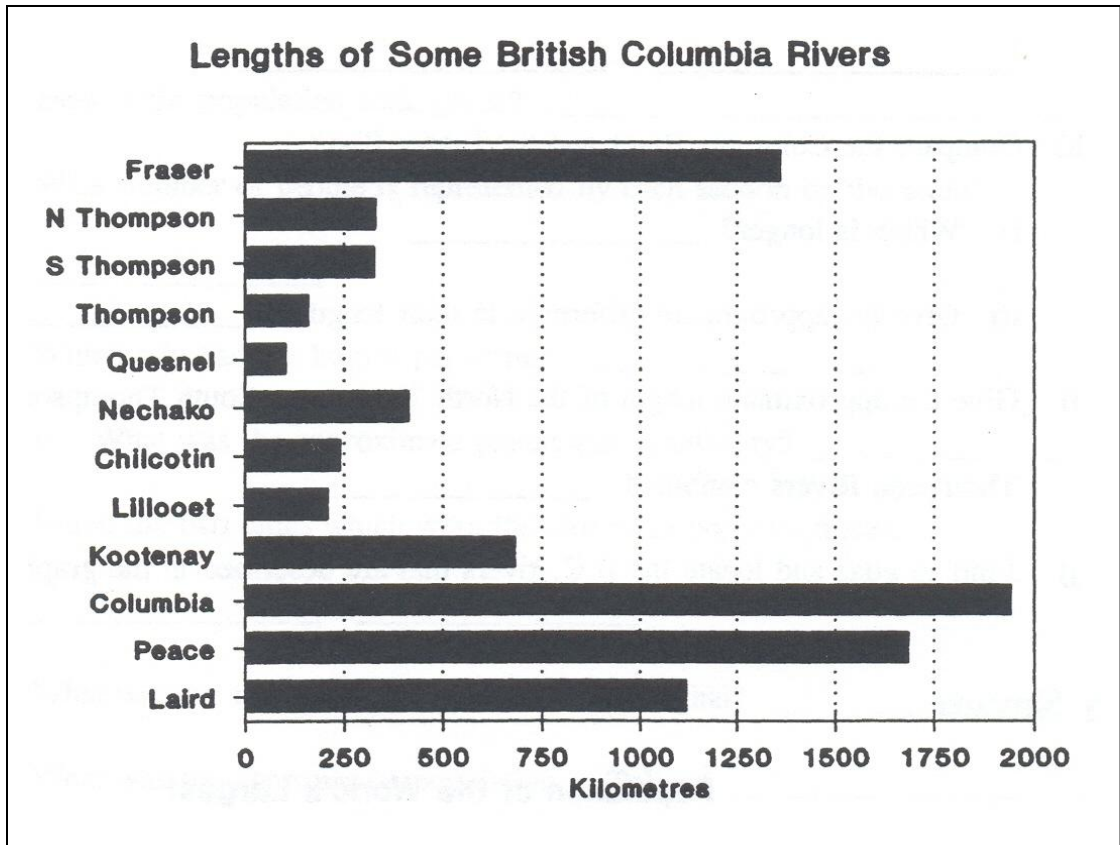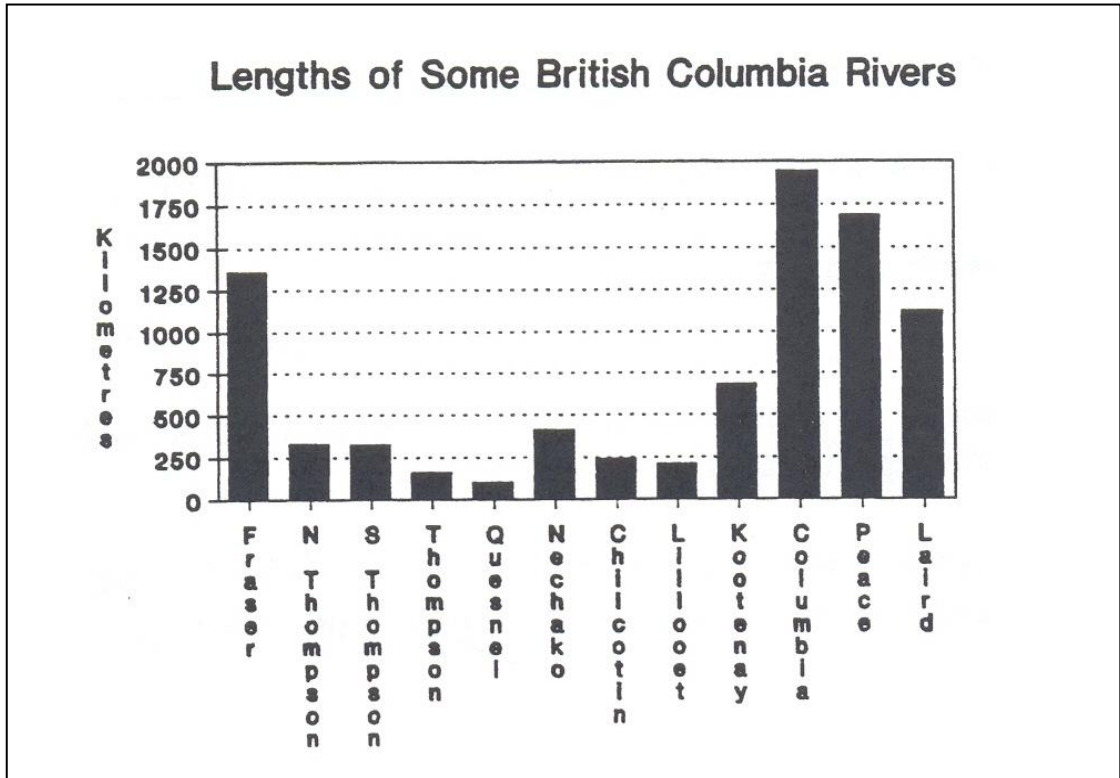
a) How many rivers are shown on this graph? _____

b) What is the title of the graph? _____

c) What is the unit of measure? _____

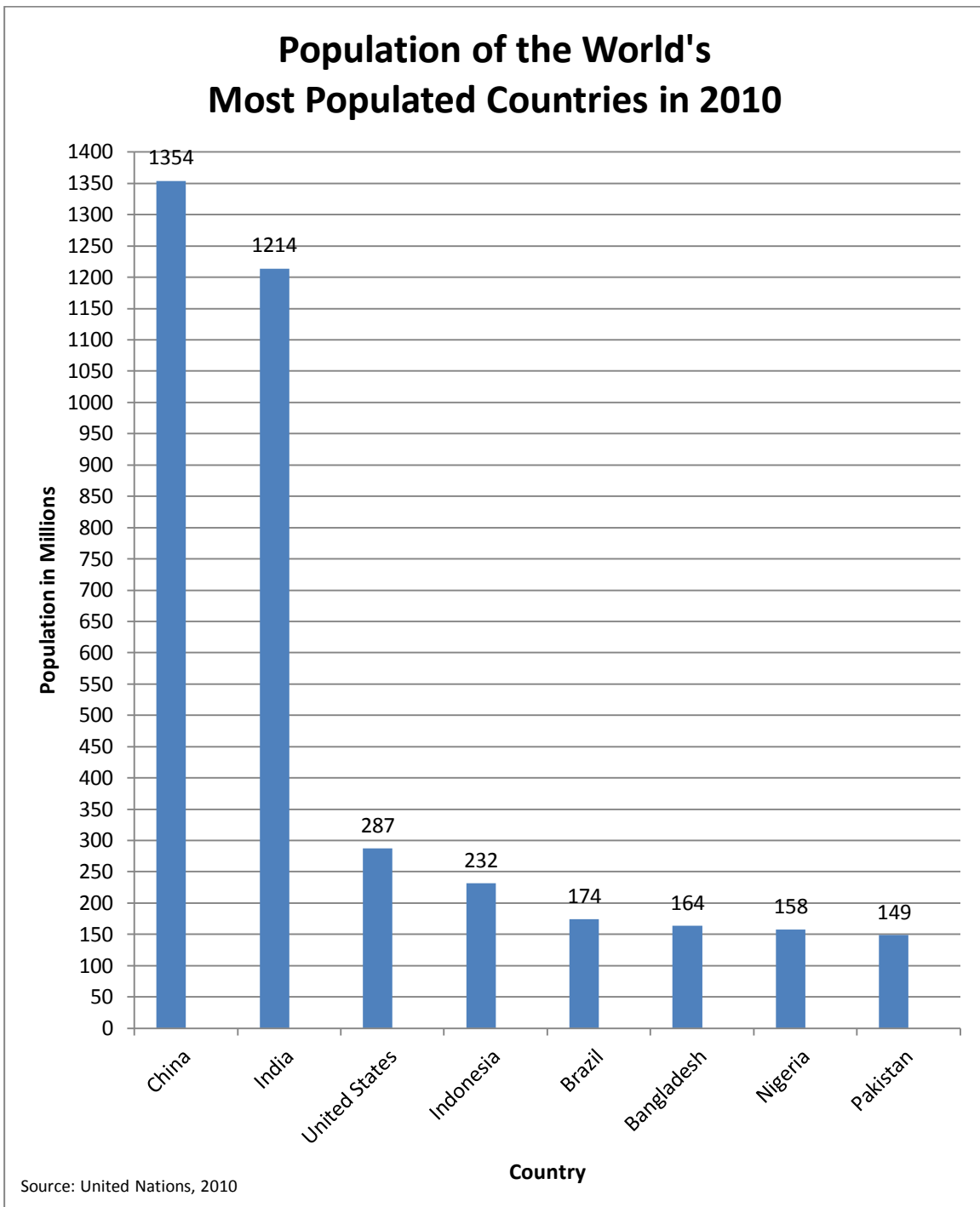d) Look at the scale for kilometres. How many kilometres are represented by each division on the page?

   _____

e) Which river is the longest? _____

   What is its length? _____

f) Which river is the shortest? _____

   What is its length? _____

g) Name two rivers which are approximately the same length.

   _____   _____

h) Compare the Columbia River and the Fraser River.

   i) Which is the longer? _____

   ii) Give the approximate difference in their lengths. _____

i) Give the approximate length of the North Thompson, South Thompson, and Thompson Rivers combined.

   _____

Answers to Graph One

a) 12 rivers   b) Lengths of some British Columbia Rivers
c) kilometres   d) 250 km   e) Columbia ≈ 1950 km
f) Quesnel ≈ 100 km   g) North Thompson & South Thompson
h) Columbia ≈ 600 km (1950 – 1350)   i) ≈ 750 km

Graph Two

Population of the World's Most Populated Countries in 2010

A bar graph with the vertical axis labeled "Population in Millions" ranging from 0 to 1400 in increments of 50, and the horizontal axis labeled "Country."

Country	Population in Millions
China	1354
India	1214
United States	287
Indonesia	232
Brazil	174
Bangladesh	164
Nigeria	158
Pakistan	149

Source: United Nations, 2010

a) Give the source of the information for this graph. _____

b) What is the unit of measure for the population scale? _____

c) What number of people is represented by each section on the scale?

_____

c) i. Which country had the largest population? _____

ii. What was the approximate population of this country? _____

e) Name the two countries which were the closest in population size.

_____   _____

f) What was the approximate population of Bangladesh? _____
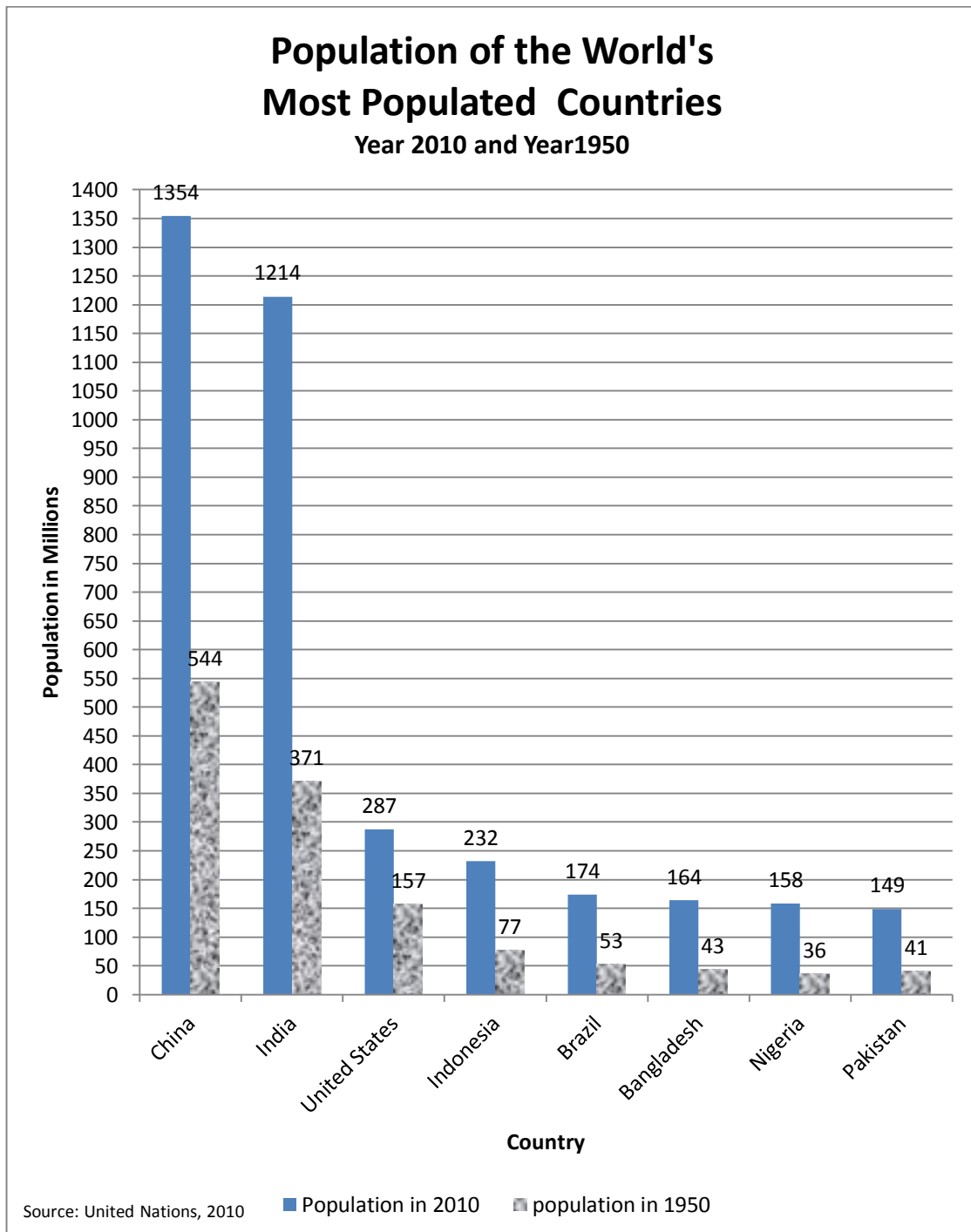
g) What was the approximate population of India? _____

Bar graphs can show more than one type of information for each item. These graphs are useful for making comparisons. The bars are usually shaded or coloured differently and a legend will be placed near the graph. The bar graphs must still all use the same unit of measure.

Graph Three

Population of the World's Most Populated Countries
Year 2010 and Year1950

Population in Millions (y-axis, 0 to 1400)

Country	Population in 2010	population in 1950
China	1354	544
India	1214	371
United States	287	157
Indonesia	232	77
Brazil	174	53
Bangladesh	164	43
Nigeria	158	36
Pakistan	149	41

Country (x-axis)

Source: United Nations, 2010   ■ Population in 2010   ▨ population in 1950

a) What is the subtitle? _____

b) Look at the legend. The grey bars give each country's population for what year?
    _____. The patterned bar gives the population

    for these same countries in what year? _____

b) What trend does the graph show? _____

d) i. Which country had the largest increase in population? (this means, which
    country's population went up by the highest number) _____

    ii. About how much was that increase? _____

e) i. Which country had the least change in population? _____
    (this means – which country's population went up the least?)
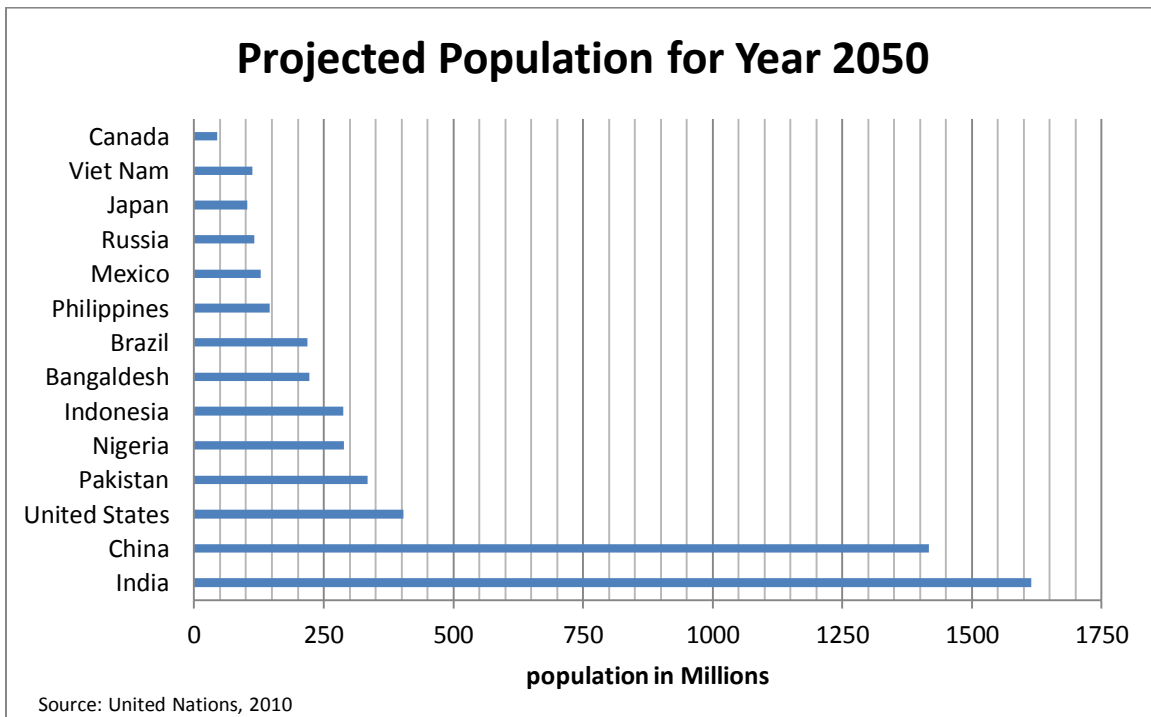
    ii. About how much was that change? _____

Topic B: Self-Test

Mark /4   **Aim 4/4**

The projected population of the world's most populated countries is shown in the following graph. Label all the parts of this graph.   **4 Marks**

1. Title
2. Axis title(s)
3. Source
4. Scale

Projected Population for Year 2050

population in Millions

Canada
Viet Nam
Japan
Russia
Mexico
Philippines
Brazil
Bangaldesh
Indonesia
Nigeria
Pakistan
United States
China
India

0   250   500   750   1000   1250   1500   1750

Source: United Nations, 2010

Answer to Self-Test Check your answers against the information in the introduction to make sure you have labeled everything right.

Topic C: Picture Graphs

Picture graphs are similar to bar graphs. **Picture graphs show comparisons between quantities.** A little picture represents a certain amount. Look for the legend to find out that amount. Picture graphs will give fractions of a picture also. For example, if the picture represents 100 things, half a picture would be 50.

Graph One

Informal Survey of Vehicles per 1000 Used in Summer Months

cars
bikes & motorcycles
large trucks
RV's & small trucks

1 picture = 100 vehicles
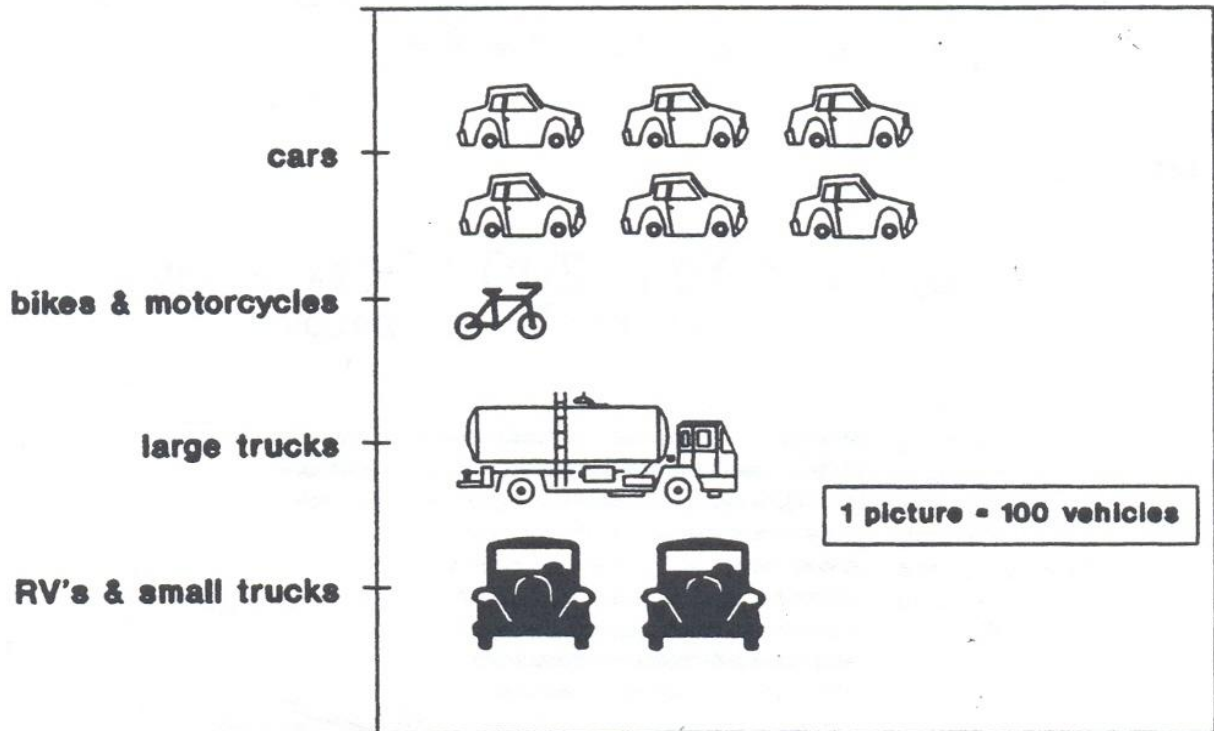
a) As a quick first impression when you look at this graph, which type of vehicle is most in use?

_____

b) What does each picture represent according to the legend? _____

c) Out of every 1 000 vehicles, how many are:

    i) cars? _____

    ii) bikes and motorcycles? _____

    iii) large trucks? _____

    iv) RVs and small trucks? _____

d) Look for other examples of picture graphs in the newspaper and in magazines. Television programs often display picture graphs to illustrate statistics.

Answers to Graph One
a) car
b) 100 vehicles
c) i) 600 ii) 100 iii) 100   iv) 200
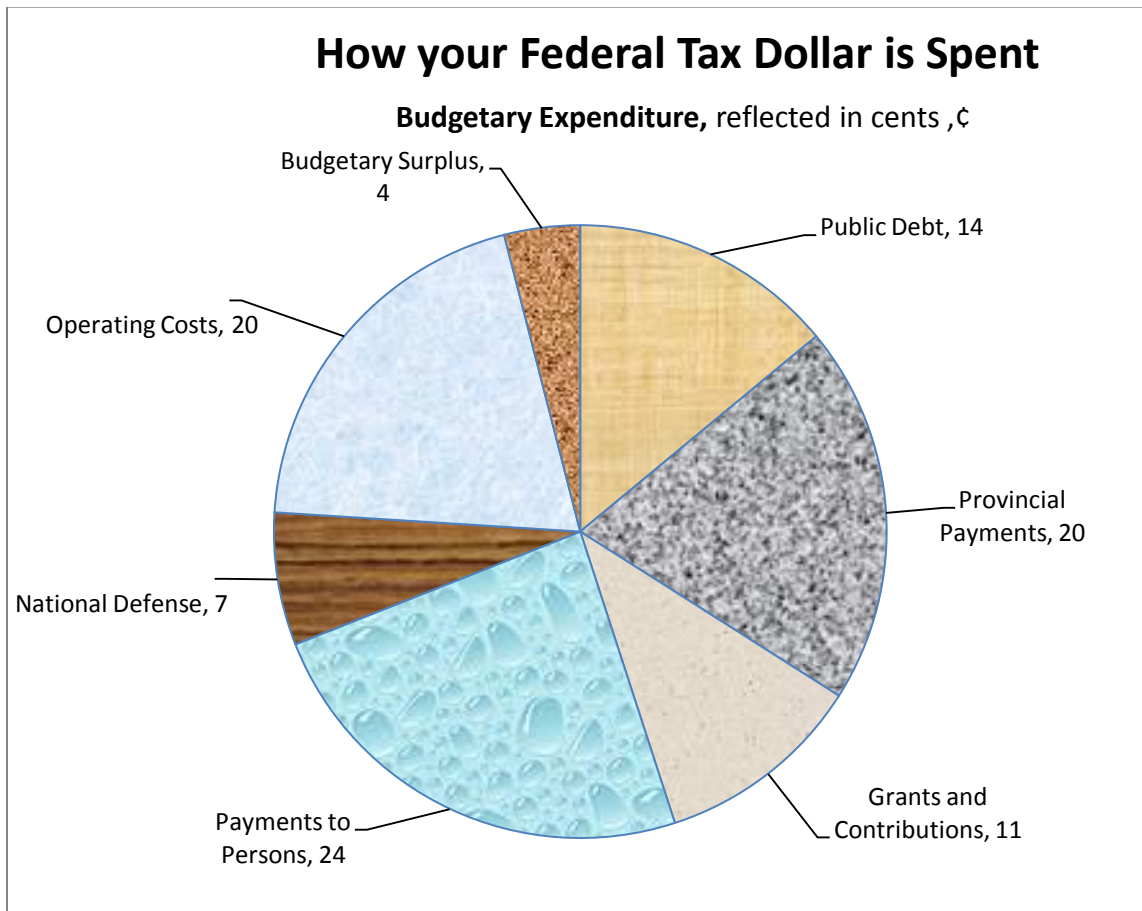
Topic D: Circle Graphs ("Pie Graphs")

Circle graphs **show how the parts of something compare to each other**. Circle graphs also **give a good picture of each part compared to the whole thing**. In a circle graph or pie graph, the complete circle is the whole thing. The parts of a circle graph may be identified with a percentage. The total of the parts must be 100%.

Graph One

The circle represents each dollar the government spends. The information for the graph was found at the Department of Finance, April 2010.

http://www.fin.gc.ca/taxdollar/09/mm-eng.asp

The parts are shown as cents of the dollar.

How your Federal Tax Dollar is Spent
Budgetary Expenditure, reflected in cents ,¢

- Budgetary Surplus, 4
- Public Debt, 14
- Operating Costs, 20
- Provincial Payments, 20
- National Defense, 7
- Grants and Contributions, 11
- Payments to Persons, 24

a) What is the biggest expense of the federal government?

_____

The federal government is the employer of hundreds of thousands of Canadians in the civil service and in the armed forces (defence budget). The *Payments to persons* shown on the graph are social security, family allowance, Canada Pension, employment insurance, and other similar services.

b) How much of each federal dollar is spent in actually operating the government business?

_____

c) What part of the federal dollar is spent on defence? _____

d) How much of each dollar is spent on Provincial Payments?_____

Write this amount as a percent. _____

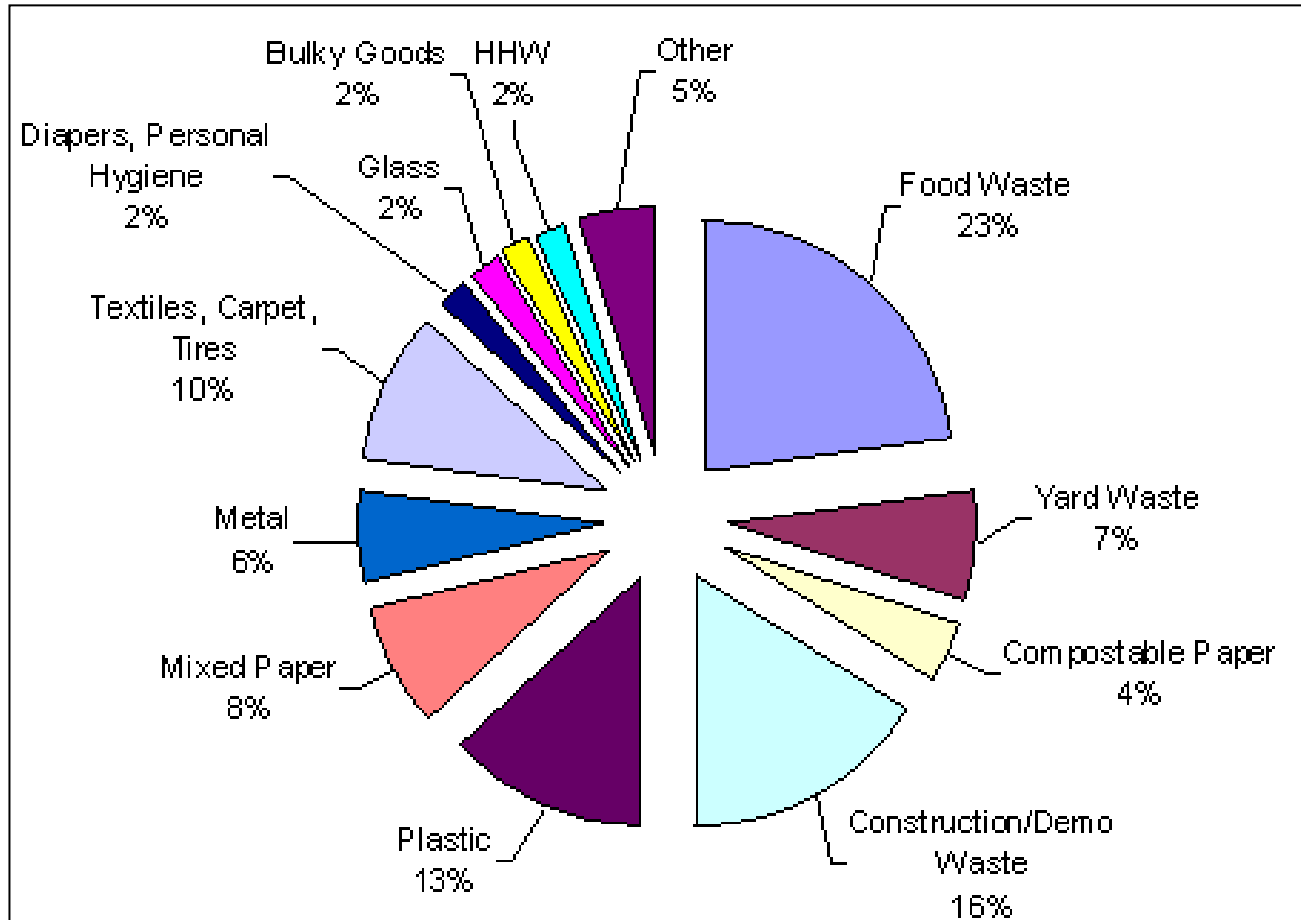e) What is the smallest expenditure of the federal government? _____

Write this amount as a percent. _____

Answers to Graph One

a) Payments to Persons
b) 20¢   c) 7¢
d) 20¢; 20%   e)   Budgetary Surplus 4%

Graph Two

2004 Nanaimo Regional Landfill
Solid Waste Composition

Source: http://www.rdn.bc.ca/cms.asp?wpID=1602

a) What makes up the largest part of the waste in the landfill site?

_____

b) What four categories contribute equal weight to the landfill site?

_____   _____

_____   _____

c) In a municipality of 139 000 people, the amount of waste going to a landfill site in one day is 150 tonnes.

    i)   What is the mass of plastics? _____

    ii) What is the mass of yard waste? _____

    iii) What is the mass of construction/Demo waste? _____

    iv) If all the food waste was composted, how many tonnes of waste would *not* end up in the landfill each day? _____

d) The plastics category can be separated into these categories:

- 6% Non-recyclable mixed plastics
- 4% film plastic
- 3% recyclable rigid food containers

If all the 3% recyclable rigid food containers were actually recycled, how many tonnes of waste would *not* end up in the landfill? _____

Answers to Graph Two

a) Food waste

b)
- Diapers, Personal Hygiene
- Glass
- Bulky Goods
- HHW (Household Hazardous Waste

c) i. 19.5 tonnes   ii. 10.5 tonnes   iii. 24 tonnes   iv. 34.5 tonnes

d) 4.5 tonnes

Topic E: Histograms

A histogram is a special bar graph that shows how a frequency (the number of times something happens) relates to a class interval (a range of numbers). **A histogram is useful when looking at how many times something happens**. It is useful to look at monthly or yearly temperatures, at test scores and groups of people based on age.

In the following graph, the height of each bar relates to how many days a temperature was between the listed temperatures in the horizontal axis.

This graph is created by counting how many days the temperatures were:

- between 0°C and -5°C (1 day)

- between -6°C and -10°C (6 days)

- between -11°C and - 15°C (8 days)

- between -16°C and - 20°C (5 days)

- between -21°C and -25°C (9 days)

- between -26°C and -30°C (2 days)

Then the information is put into graph form.

Graph One

**Daily Minimum Temperatures
in the Month of January, 2010**

Atlin, BC

Source: Environment Canada
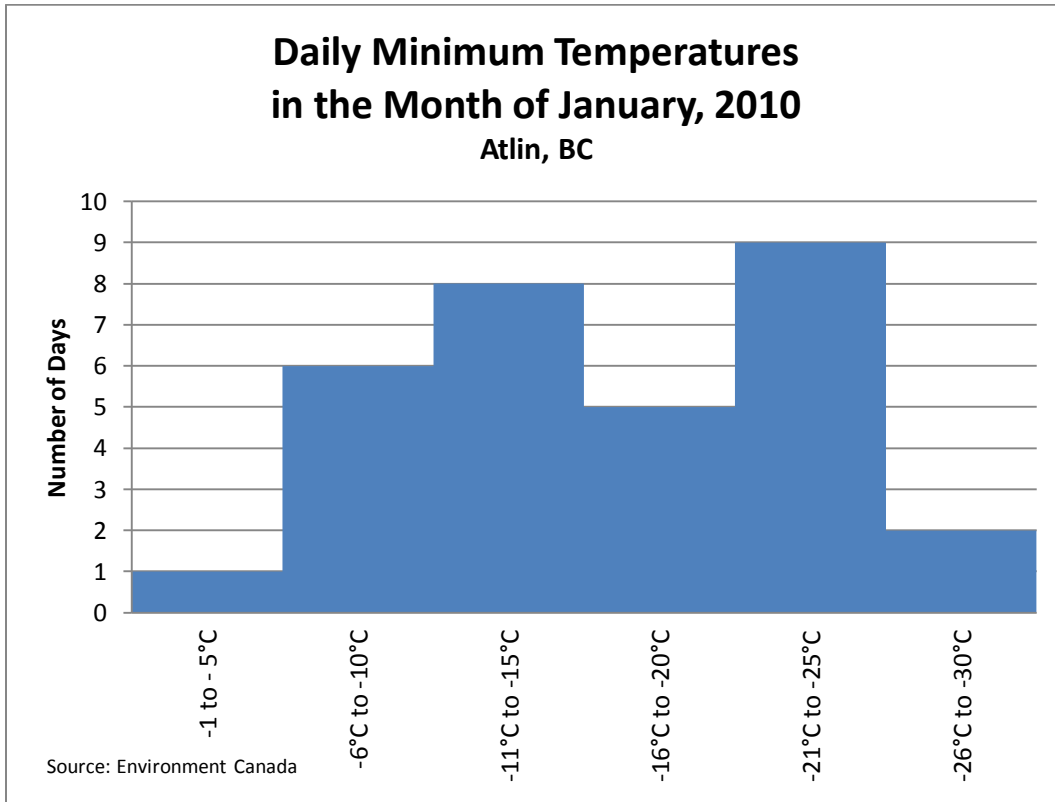
a) How many degrees in temperature change is in each bar? _____

b) What is the source of the information? _____

c) Which temperature was the most common in the month of January?_____

d) Which community does this graph represent? _____

e) Which temperature was felt the *least* in Atlin in January, 2010?_____

Answers to Graph One

a) five degrees   b) Environment Canada   c) -21°C to - 25°C

d) Atlin, BC   e) -1°C to -5°C

Graph Two

In this Fundamental Math course, students' marks were collected from the final test. The graph shows the results.
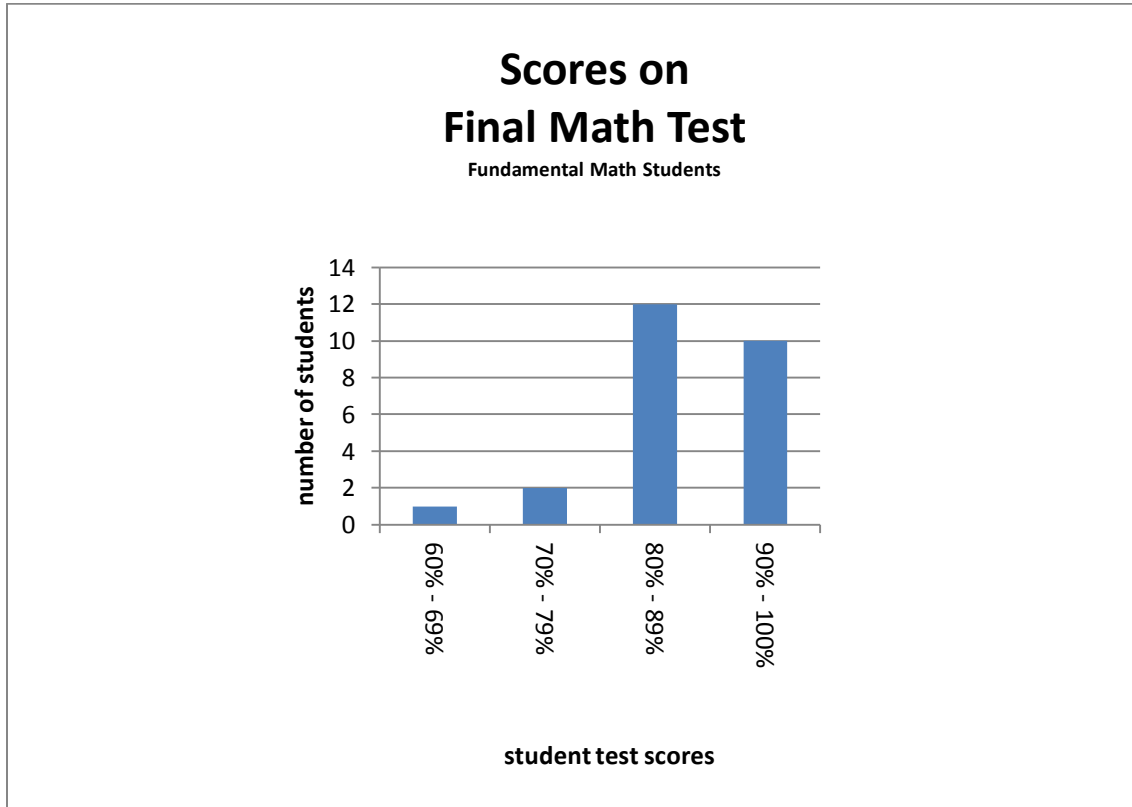
a) How many students took the test? _____

b) How many students scored 80% to 89% on the test? _____

c) In order to pass, students must get 80% or over on the test. How many students will have to re-write the test? _____

d) How many more students got 80% to 89% than 90% to 100%? _____

Answers to Graph Two

a) 25   b) 12   c) 3   d) 2

Topic F: Tables

Tables are an everyday way of **organizing information, or one's own work**.

Graph One

The following table is from the BC Ferries website:

http://www.bcferries.com/schedules/mainland/copr-current.html.

It shows the departure times from each community.

Comox - Powell River (Little River-Westview)	
Crossing Time: 1 hour 20 minutes	
Distance: 17 nautical miles	
Leave Comox (Little River)	**Leave Powell River (Westview)**
6:30 am *Daily except Dec 25 and Jan 1*	8:10 am *Daily except Dec 25 and Jan 1*
10:10 am	12:00 pm
3:15 am	5:15 pm
7:15 am	8:45 pm

a) How many ferry runs go from Powell River to Comox each day? _____

b) On what days does the ferry not run at 6:30 am and 8:10 am? _____

_____

c) How long is a crossing time? _____

d) When will the first ferry from Comox arrive in Powell River? _____

e) When will the last ferry from Powell River arrive in Comox? _____

f) How many nautical miles is covered by the ferry's course? _____

Graph Two

A cereal recipe explains the quantities to use when making hot cereal.

Servings	Salt	Water	Cereal
1	¼ tsp	1.5 cups	¼ cups
4	1 tsp	6 cups	1 cup

a) How much salt should be put in for one serving? _____

b) How much water is to be used when making 4 servings of cereal? _____

c) How many cups of cereal should be used for making 1 serving of cereal? _____

d) To make two servings of cereal, how much water is needed? _____

e) To make eight servings of cereal, how much cereal should be used? _____

f) To make ten servings of cereal, how much salt should be used? _____

Review

Refer back to each lesson on graphs and explain when to use each type of graph.

Line Graph: _____

_____

Bar Graph: _____

_____

Picture Graph: _____

_____

Circle Graph: _____

_____

Histogram: _____

_____

Table: _____

_____

TEST TIME!

Ask your instructor for the
Practice Test for this unit.

Once you've done the practice test,
you need to do the unit 5 test.

Now that you have completed the whole book,
you will need to do the final test.

Please see your instructor
for the practice test and the final test.

Good Luck!

Book Six Final Review

You will now practice all the skills you learned in Book 6. You can use this as a review for your final test.

If you can't remember how to do a question, go back to the lesson on this topic to refresh your memory. The unit and topic for where each question came from is listed next to the question.

> Example: **1A** means Unit 1, Topic A

1-A

1. **Write the ratios asked for.**

   a) Lillian biked for 6 hours, and covered a total of 45 km. What is the ratio of kilometers to hours?

      _____

   b) Nine hundred cars were lined up at the ferry terminal. 300 hundred cars got on the next sailing. Write a ratio of how many cars were left behind to how many cars got on the first sailing.

      _____

1 – B

2. **Simplify these ratios.**

   a) 9:12 = _____   b) 50:5 = _____

   c) 56:7 = _____   d) 100:120 = _____

3. **Write the following ratios as rates**

   a) 110 kilometres to 2 hours   _____

   b) 9 cups of flour to 3 tablespoons of yeast   _____

   c) $5.98 to 3 kilograms of apples   _____

   d) 240 000 people to 300 square kilometers   _____

1 – C

4. Solve these proportions.

    a) $1:3 = N:12$ b) $25:N = 20:4$

    c) $N:49 = 14:98$ d) $4\frac{1}{2}:6 = N:3.6$

    e) 2 out of 5 college students receive financial aid. North Island College has 8500 students, how many receive financial aid?

    f) The dose for cough syrup is 20 millilitres for each 100 pounds of body weight. How much should be given to a 34 pound child? Round to the nearest millilitre.

2 – A

5. Write these percents using numerals and the percent sign.

    a) Seventy two percent   _____

    b) Three - fourths percent   _____

    c) one hundred two percent   _____

6. Write these percents in words.

    a) 12 %   _____

    b) 35.7 %   _____

    c) $\frac{1}{5}$ %   _____

7. Change the percents to equivalent decimals.

a) 17 % _____

b) 75 % _____

c) 10.35 % _____

d) $98\dfrac{1}{2}$ % _____

e) $\dfrac{1}{3}$ % _____

f) 304 % _____

8. Write the decimals as percents.

a) 0.45 _____

b) 4.75 _____

c) 0.34 _____

d) 0.099 _____

9. Change each percent to an equivalent common fraction. Put the fraction in lowest terms.

a) 45 % _____

b) $33\dfrac{1}{3}$ % _____

c) 14 % _____

d) 3 % _____

e) 20 % _____

f) 250 % _____

10. Write the percent equivalent.

a) $\dfrac{1}{5}$ _____

b) $\dfrac{2}{3}$ _____

c) $\dfrac{1}{4}$ _____

d) $\dfrac{3}{8}$ _____

3 – A

11. Find the answers.

(Express percents rounded to the nearest tenth, money to the nearest cent and decimals to the nearest thousandth. Please show all your work. Use proportion.)

    a) 13% of 52 =   _____

    b) $17\frac{1}{2}$% of 1040 is   _____

    c) $\frac{9}{10}$% of 2400 is   _____

    d) What is 15% of 160?   _____

    e) What is 135% of 1080?   _____

12.   **Solve these problems. Be sure to show all your work.**

    a) Marianne is renovating her kitchen, and she is ordering everything from her local hardware store. She is getting a sink for 204.79, a dishwasher for $524.95, a counter for $949.99, flooring for $719.95, and a fridge for $579.49.

        a) Calculate the HST (12%)

        ii) Calculate the total cost, including the taxes.

b) Shane sold a home for $340 500.00 for a client. She earned 6% commission. How much money did Shane make?

c) A love seat is originally priced at $904.00, it is offered at 45% off. What is the discount price?

d) Calculate the total cost in Canadian dollars of this purchase made in the United States. Assume $1.00 Canadian = $0.92 U.S.

Clothes – Total price in $US = **$317.98.**

    i. Price in Canadian dollars   _____

    ii. Duty at 13.5%   _____

    iii. Total of Canadian value + duty   _____

    iv. HST (12%) on Canadian value + duty   _____

    v. Total cost in Canadian dollars   _____

13.   **Find the answers.**

a) 34 is what percent of 85?   _____

b) 9 is what percent of 1125?   _____

c) What % of 150 is 114?   _____

d) What % of 68 is 170?   _____

e) $125\frac{1}{2}$ is what percent of 25?   _____

f) $33\frac{1}{3}$ % of what number is 60?   _____

g) 32 is 20% of what number?   _____

h) 75% of what number is 675?   _____

i) 45% of what number is 270?   _____

j) 3.75 is $1\frac{1}{4}$ % of?   _____

14.   Solve these problems. Be sure to show all your work.

a) A printer is priced at $399. It is marked 30% off.
   i) what is the sale price of the printer?

   ii) What is the sale price of this printer with HST (12%)?

b) Joseph bought a second hand car that was originally priced at $6,500. He got the car for $6 350.50. What was the percent of savings for Freddie?

c) Betty Ann got 84 out of 95 on her English test. What percent did she get on the test? (Round your answer to the nearest tenth of a percent.)

d) The Vancouver Fire and Rescue Service has 797 uniformed personnel, about 0.8% are women. About how many women are in uniform in the Vancouver Fire and Rescue Service?

e) Jake is a computer salesperson. He receives a monthly salary of $1055 plus 15% on all his sales over $5 500. What was his total monthly earnings if his sales were $12 400 in one month?

f) In a recent survey of 1250 British Columbians, 461 rated their health as excellent. What percent of these British Columbians rate their health as excellent?

g) The local ski hill sold 3800 season's passes in 2009. The 2010 sales are down 10.5%.

Find the number of season's passes sold in 2010.

5 - B

15. Line graph

Average Temperatures for Smithers, BC
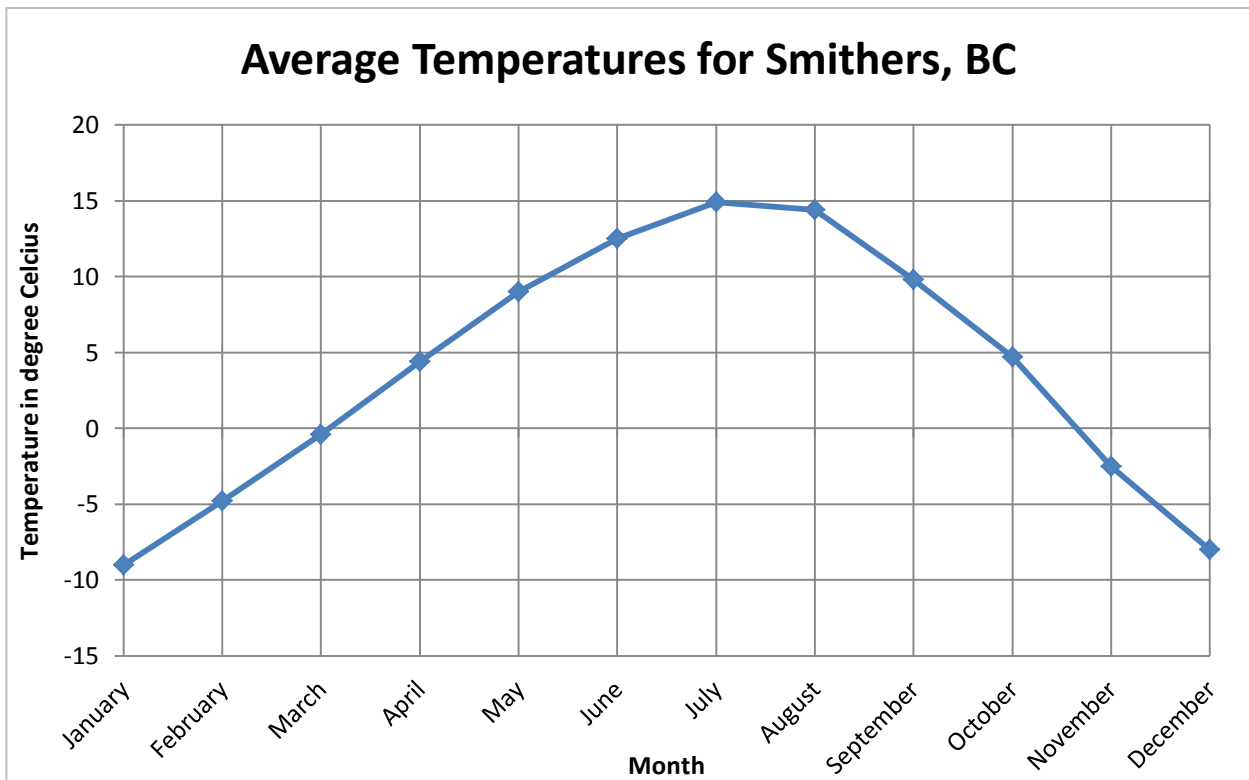
a) Which month has the highest temperature in Smithers? _____

b) Which month has the lowest temperature in Smithers? _____

c) Between the months of January to July, is there an increase or decrease in temperature?

_____

d) What is the difference between the monthly temperature for August and the monthly temperature for October? _____

e) What is the title of this graph? _____

f) What is the trend of the temperature in Smithers? _____

16. Bar Graph

Highest Mountains in British Columbia

Height in metres (y-axis, scale 0 to 5000)

Mountains (x-axis):
- Fairweather Mountain
- Moutn Quincy Adams
- Mount Waddington
- Mount Robson
- Mount Root
- Mount Tiedemann
- Combatant Mountain
- Mount Columbia
- Asperity Mountain

source:
Statistics
Canada,
2010
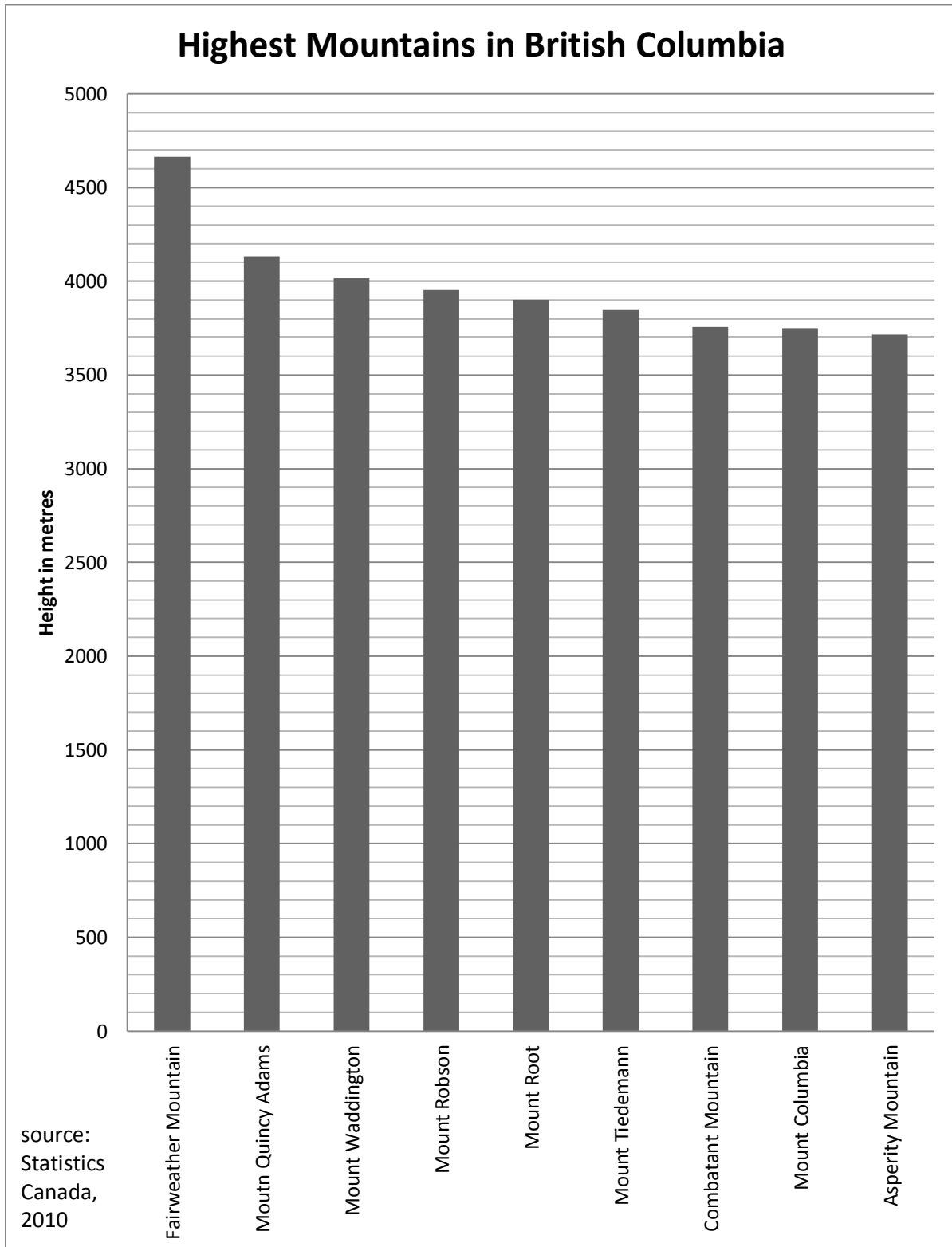
a) What is the name of the tallest mountain in BC? _____

b) What is the height of the tallest mountain in BC? _____

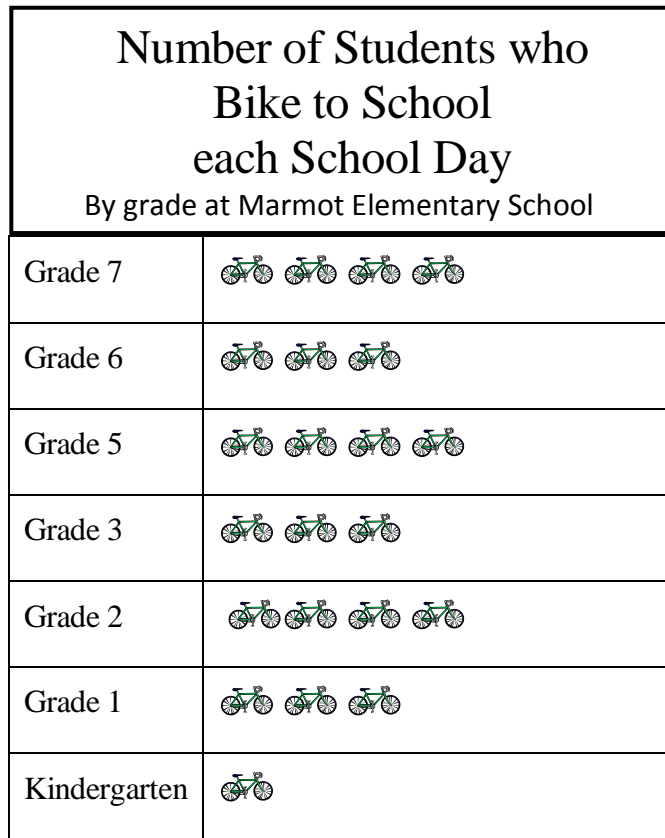c) How many mountains are over 4000 metres and under 5000 metres in height?

   _____

d) Which two mountains in this chart are very similar in height?

   _____   _____

e) What is the difference (approximately) in height between Fairweather Mountain and Asperity Mountain? _____

f) What is the difference (approximately) in height between Mount Quincy Adams and Combatant Mountain? _____

g) Mount Everest is 8848 metres tall. How much taller is Mount Everest than BC's tallest mountain? _____

17. Picture Graph

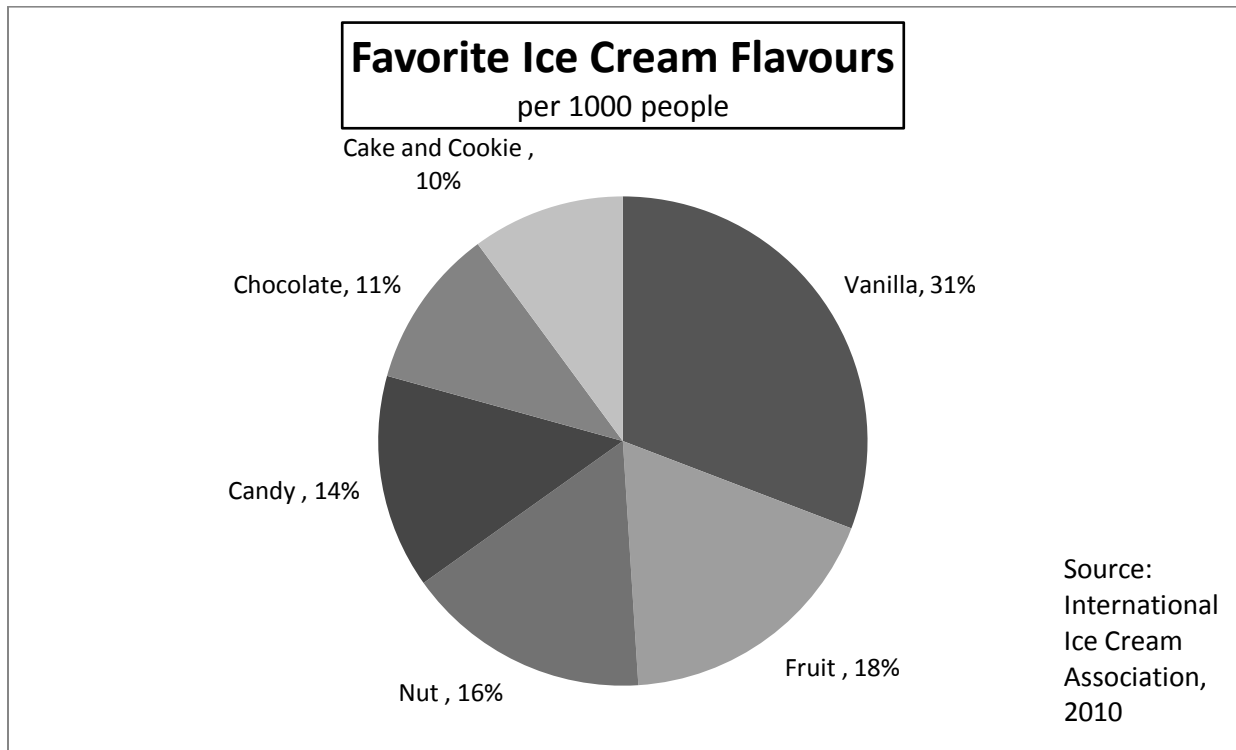| Number of Students who Bike to School each School Day — By grade at Marmot Elementary School	
Grade 7	🚲 🚲 🚲 🚲
Grade 6	🚲 🚲 🚲
Grade 5	🚲 🚲 🚲 🚲
Grade 3	🚲 🚲 🚲
Grade 2	🚲 🚲 🚲 🚲
Grade 1	🚲 🚲 🚲
Kindergarten	🚲

🚲 = 5 bikers

Total School Population = 563

a) How many students in grade 2 ride bikes to school? _____

b) Which classes have the most bikers? _____

c) Which class has the least bikers? _____

d) How many more bikers are in grade 7 than kindergarten? _____

e) How many students bike in total? _____

f) What percent of the school bikes each day? _____

18. Circle Graph

Favorite Ice Cream Flavours
per 1000 people

Cake and Cookie , 10%

Chocolate, 11%

Vanilla, 31%

Candy , 14%

Nut , 16%

Fruit , 18%

Source: International Ice Cream Association, 2010

a) Which is the most favourite ice cream flavour? _____

b) Which is the least liked ice cream flavour? _____

c) How many people (out of 1000) like fruit flavoured ice cream? _____

d) How many more people like vanilla over chocolate? _____

19. Histogram

Annual Earnings at Local Discount Store
(part time employees)
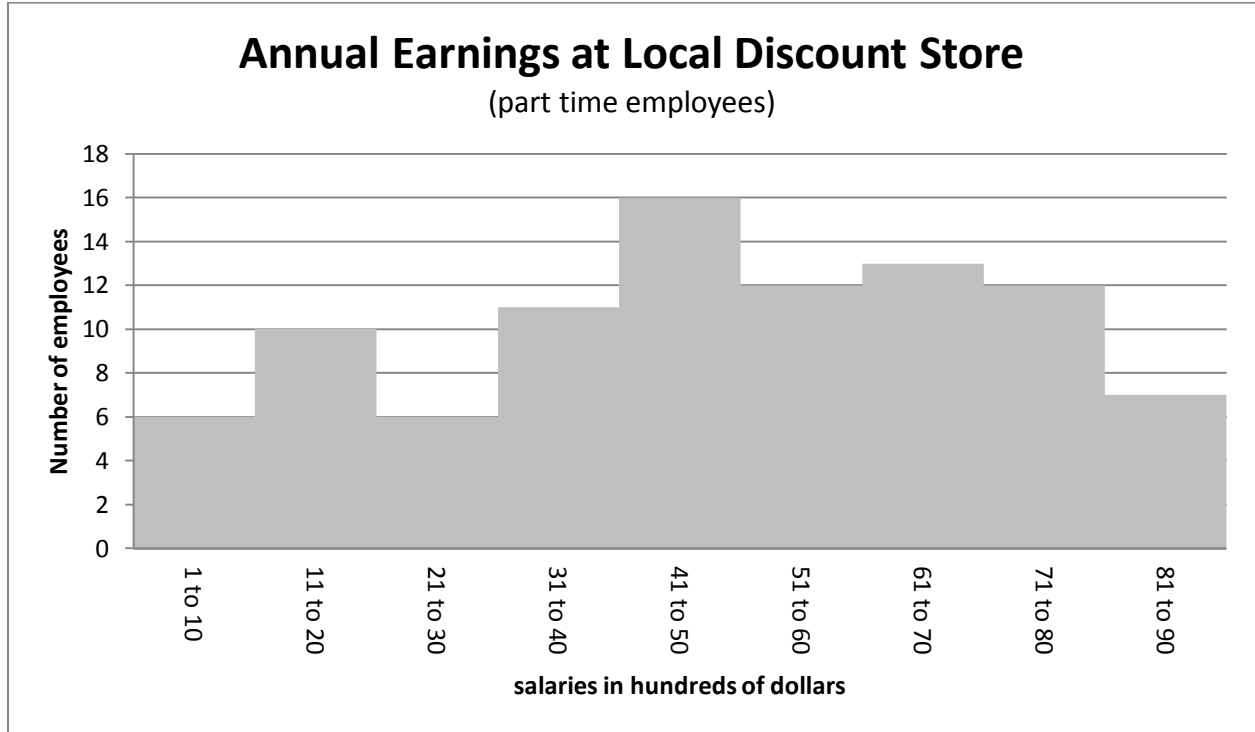
a) What number of employees earn between $3100 to $4000._____

b) How many employees earn between $8100 and $9000 in a year at the local discount

   store? _____

c) The greatest number of employees are in which earnings group?_____

d) How many employees are in that group? _____

e) How many employees earn $5000 or less?_____

20. Table

Tide chart for Alert Bay

Date	Time	Tide (in metres)	High or Low?
2010-10-19	00:22	3.83	High Tide
2010-10-19	06:19	1.52	Low Tide
2010-10-19	12:26	4.22	High Tide
2010-10-19	18:53	1.29	Low Tide

a) What is the date for this tide chart?

b) Where is this tide chart for?

c) What are the two low tide times?

d) What are the two times for high tides?

e) Which is the highest tide of the day?

f) What is the difference between the highest tide and lowest tide of the day?

Answers to Book 6 Final Review

1. a) 45:6   b) 600:300

2. a) 3:4   b) 10:1   c) 8:1   d) 5:6

3. a) 55 km/hr or 55km:1 hr   b) 3 cups of flour to 1tbsp yeast

    c) $1.99 to 1 kg apples   d) 800 people to 1 square km

4. a) 4   b) 5   c) 7   d) 2.7   e)3400   f) 136 ml

5. a) 72%   b) $\frac{3}{4}$%   c) 102%

6. a) twelve percent b) thirty five point seven percent   c) one fifth percent

7. a) 0.17   b) 0.75   c) 0.1035   d) 0.985   e) $0.00\bar{3}$ f) 3.04

8. a) 45%   b) 475%   c) 34%   d) 9.9%

9. a) $\frac{9}{20}$ b) $\frac{1}{3}$ c) $\frac{7}{50}$ d) $\frac{3}{100}$ e) $\frac{1}{5}$ f) $2\frac{1}{2}$

10. a) 20%   b) $66.\bar{6}$% or $66\frac{2}{3}$%   c) 25%   d) 37.5%

11. a) 6.76   b) 182   c) 21.6   d) 24   e) 1458

12. a) i) $357.50   ii) $3336.67   b) $20 430.00   c) $497.20   d) i. $345.63   ii. $46.66
    iii. $392.29   iv. $47.07   v. $439.36

13. a) 40   b) 0.8   c) 76   d) 250   e) 502   f) 20

    g) 160   h) 900   i) 600   j) 300

14. a) i) $279.30   ii) $312.82   b) 2.3%   c) 88.4%   d) 6   e) $2090

    f) 36.88%   g) 3401

15. a) July   b) January   c) increase   d) approximately 9 degrees

    e) Average Temperature for Smithers, BC

    f) the temperature goes up from January to July, and goes down from July to December

16. a) Fairweather Mountain   b) (approx) 4650 metres   c) 3

    d) Combatant Mountain and Mount Columbia   e) 950 metres

    f) 370 metres   g) 4198 metres

17. a) 20   b) grades 2, 5, 7   c) kindergarten   d) 15

    e) 484   f) 86%

18. a) vanilla   b) cake and cookie   c) 180   d) 20%

19. a) 11   b) 7   c) $4100 to $5000   d) 16   e) 49

20. a) October 19, 2010   b) Alert Bay   c) 6:19 and 18:53

    d) 00:22 and 12:26   e) 4.22 metres   f) 2.93 metres

Glossary

addends The numbers to be added together in an addition question. In $3 + 5 = 8$, the addends are 3 and 5.

axis Any straight line used for measuring or as a reference.

balance Balance has many meanings. In money matters, the balance is the amount left. It might be the amount left in a bank account (bank balance) or it might be the amount you still must pay on a bill (balance owing).

cancelled cheque A cheque that has been cashed. The cheque is stamped, or cancelled, so it is no longer negotiable.

circumference The distance around a circle; the perimeter of a circle.

commission Salespeople may be paid a percentage of the money made in sales. The commission is part or all their earnings.

common fractions eg, $\frac{2}{3}, \frac{3}{7}, \frac{49}{50}$

cross multiply In a proportion, multiply the numerator of the first fraction times the denominator of the second fraction. Then multiply the denominator of the first fraction times the numerator of the second fraction. In a true proportion, the products of the cross multiplication are equal.

denominator The bottom number in a common fraction; tells into how many equal parts the whole thing has been divided.

diameter The distance across a circle through its centre.

difference The result of a subtraction question, the answer. Subtraction gives the difference between two numbers.

digit Any of the ten numerals (0 to 9) are digits. This term comes from our ten fingers which are called digits. The numerals came to be called "digits" from the practice of counting on the fingers!

discount An amount taken off the regular cost. If something is bought "at a discount" it is bought at less than the regular price.

divide To separate into equal parts.

dividend The number or quantity to be divided; what you start with before you divide.

divisor The number of groups or the quantity into which a number (the dividend) is to be separated.

equal = The same as

equation A mathematical statement that two quantities are equal. An equation may use numerals with a letter to stand for an unknown quantity. $6 + Y = 9$

equivalent Equal in value; equivalent numbers (whole or fractions) can be used interchangeably; that is, they can be used instead of each other.

estimate Make an approximate answer. Use the sign $\approx$ to mean approximately equal.

factors The numbers or quantities that are multiplied together to form a given product. $5 \times 2 = 10$, so 5 and 2 are factors of 10.

fraction Part of the whole; a quantity less than one unit.

horizontal in a flat position; we are horizontal when we lie in a bed. A horizontal line goes across the page.

improper fraction A common fraction with a value equal to or more than one.

infinite Without end, without limit.

invert To turn upside down.

like fractions With the same denominators.

lowest terms When the terms of a common fraction or ratio do not have a common factor (except 1), the fraction or ratio are in lowest terms (also called simplest form).

minuend The first number in a subtraction question.

mixed number A whole number and a common fraction. $1\dfrac{3}{4}$

mixed decimal A whole number and a decimal fraction.   1.75

multiple If a certain number is multiplied by another number, the product is a multiple of the numbers. Think of the multiplication tables. For example, 2, 4, 6, 8, 10, 12, 14...are multiples of 2.

multiplicand The number to be multiplied.

multiplier The number you multiply by.

negotiable Something which can be cashed, that is, exchanged or traded as money.

numbers Numbers represent the amount, the place in a sequence; *number* is the idea of quantity or order.

numerals The digits 1,2,3,4,5,6,7,8,9,0 are also called numerals. These ten digits are combined to make infinite numerals. Digits are like the letters, numerals are like the words and numbers are the meaning.

numerator The top number in a common fraction; the numerator tells how many parts of the whole thing are being considered.

overdrawn If the value of the cheques or money taken from a bank account is higher than the amount of money in the account, then the account is overdrawn. The account is "in the hole" or "in the red" are expressions sometimes used.

parallel Two objects or lines side by side, never crossing and always the same distance from each other. Railway tracks are parallel, the lines on writing paper are parallel.

percent % For every one hundred.

perimeter The distance around the outside of a shape.

place value We understand numbers by the way the digits (numerals) are arranged in relationship to each other and to the decimal point. Each position has a certain value. Our number system is a **decimal system**. The place value is based on **ten.**

prime number A number that can only be divided evenly by itself and 1.

product The result of a multiplying question, the answer.

proper fraction A common fraction with a value less than one.

proportion Generally, proportion is a way of comparing a part of something to the whole thing. Eg. his feet are small in proportion to his height. In mathematics, proportion is used to describe two or more ratios that are equivalent to each other.

quotient The result of a division question; the quotient tells how many times one number is contained in the other.

radius The distance from the centre of a circle to the outside of the circle.

ratio The relationship between two or more quantities. Eg. the ratio of men to women in the armed forces is 10 to 3 (10:3)

---

reciprocal A number, when multiplied by its reciprocal, equals 1. To find the reciprocal of a common fraction, invert it. $\frac{3}{5} \times \frac{5}{3} = 1$

reduce Write a common fraction in lowest terms. Divide both terms by same factor.

remainder The amount left when a divisor does not divide evenly into the dividend. The remainder must be less than the divisor.

sign In mathematics, a symbol that tells what operation is to be performed or what the relationship is between the numbers.
- $+$ plus, means to add
- $-$ minus, means to subtract
- $\times$ multiplied by, "times"
- $\div$ divided by, division
- $=$ equal, the same quantity as
- $\neq$ not equal
- $\approx$ approximately equal
- $<$ less than
- $>$ greater than
- $\leq$ less than or equal to
- $\geq$ greater than or equal to

simplify See *reduce*.

subtrahend The amount that is taken away in a subtraction question.

sum The result of an addition question, the answer to an addition question.

symbol A written or printed mark, letter, abbreviation etc. that stands for something else.

term a) A definite period of time, such as a school term or the term of a loan.
b) Conditions of a contract; the terms of the agreement. c) In mathematics, the quantities in a fraction and in a ratio are called the *terms* of the fraction or the *terms* of the ratio. In an algebra equation, the quantities connected by a + or - sign are also called terms.

total The amount altogether.

transaction One piece of business. A transaction often involves money. When you pay a bill, take money from the bank or write a cheque, you have made a transaction.

unit Any fixed quantity, amount, distance or measure that is used as a standard. In mathematics, always identify the unit with which you are working. Eg. 3 km, 4 cups, 12 people, $76, 70 books, 545 g.

unit price The price for a set amount. Eg. price per litre, price per gram.

unlike fractions Fractions which have different denominators.

vertical in an up and down position; we are vertical when we are standing up. On a page, a vertical line is shown from the top to the bottom of the page.